of one body is that which begins another. 3, that which is not part of any body is nothing. Nothing is that which fills no space.

If one single point placed in a circle may be the starting point of an infinite number of lines, and the termination of an infinite number of lines, there must be an infinite number of points separable from this point, and these when reunited become one again; whence it follows that the part may be equal to the whole.

46.

The point, being indivisible, occupies no space. That which occupies no space is nothing. The limiting surface of one thing is the beginning of another. 2. That which is no part of any body is called nothing. 1. That which has no limitations, has no form. The limitations of two conterminous bodies are interchangeably the surface of each. All the surfaces of a body are not parts of that body.

Of the line (47-48).

47.

DEFINITION OF THE NATURE OF THE LINE.

The line has in itself neither matter nor substance and may rather

be called an imaginary idea than a real object; and this being its nature it occupies no space. Therefore an infinite number of lines may be conceived of as intersecting each other at a point, which has no dimensions and is only of the thickness (if thickness it may be called) of one single line.

HOW WE MAY CONCLUDE THAT A SUPERFICIES TERMINATES IN A POINT?

An angular surface is reduced to a point where it terminates in an angle. Or, if the sides of that angle are produced in a straight line, then--beyond that angle--another surface is generated, smaller, or equal to, or larger than the first.

48.

OF DRAWING OUTLINE.

Consider with the greatest care the form of the outlines of every object, and the character of their undulations. And these undulations must be separately studied, as to whether the curves are composed of arched convexities or angular concavities.

49.

The nature of the outline.

The boundaries of bodies are the least of all things. The proposition is proved to be true, because the boundary of a thing is a surface, which is not part of the body contained within that surface; nor is it part of the air surrounding that body, but is the medium interposted between the air and the body, as is proved in its place. But the lateral boundaries of these bodies is the line forming the boundary of the surface, which line is of invisible thickness. Wherefore O painter! do not surround your bodies with lines, and above all when representing objects smaller than nature; for not only will their external outlines become indistinct, but their parts will be invisible from distance.

50.

Definition of Perspective.

[Drawing is based upon perspective, which is nothing else than a thorough knowledge of the function of the eye. And this function simply consists in receiving in a pyramid the forms and colours of all the objects placed before it. I say in a pyramid, because there is no object so small that it will not be larger than the spot where these pyramids are received into the eye. Therefore, if you extend the lines from the edges of each body as they converge you will bring them to a single point, and necessarily the said lines must form a pyramid.]

[Perspective is nothing more than a rational demonstration applied to the consideration of how objects in front of the eye transmit their image to it, by means of a pyramid of lines. The Pyramid is the name I apply to the lines which, starting from the surface and edges of each object, converge from a distance and meet in a single point.]

[Perspective is a rational demonstration, by which we may practically and clearly understand how objects transmit their own image, by lines forming a Pyramid (centred) in the eye.]

Perspective is a rational demonstration by which experience confirms that every object sends its image to the eye by a pyramid of lines; and bodies of equal size will result in a pyramid of larger or smaller size, according to the difference in their distance, one from the other. By a pyramid of lines I mean those which start from the surface and edges of bodies, and, converging from a distance meet in a single point. A point is said to be that which [having no dimensions] cannot be divided, and this point placed in the eye receives all the points of the cone.

[Footnote: 50. 1-5. Compare with this the Proem. No. 21. The paragraphs placed in brackets: lines 1-9, 10-14, and 17--20, are evidently mere sketches and, as such, were cancelled by the writer; but they serve as a commentary on the final paragraph, lines 22-29.]

51.

IN WHAT WAY THE EYE SEES OBJECTS PLACED IN FRONT OF IT.

The perception of the object depends on the direction of the eye.

Supposing that the ball figured above is the ball of the eye and let the small portion of the ball which is cut off by the line s t be the pupil and all the objects mirrored on the centre of the face of the eye, by means of the pupil, pass on at once and enter the pupil, passing through the crystalline humour, which does not interfere in the pupil with the things seen by means of the light. And the pupil having received the objects, by means of the light, immediately refers them and transmits them to the intellect by the line a b. And you must know that the pupil transmits nothing perfectly to the intellect or common sense excepting when the objects presented to it by means of light, reach it by the line a b; as, for instance, by the line b c. For although the lines m n and f g may be seen by the pupil they are not perfectly taken in, because they do not coincide with the line a b. And the proof is this: If the eye, shown above, wants to count the letters placed in front, the eye will be obliged to turn from letter to letter, because it cannot discern them unless they lie in the line a b; as, for instance, in the line a c. All visible objects reach the eye by the lines of a pyramid, and the point of the pyramid is the apex and centre of it, in the centre of the pupil, as figured above.

[Footnote: 51. In this problem the eye is conceived of as fixed and immovable; this is plain from line 11.]

Experimental proof of the existence of the pyramid of sight (52-55).

52.

Perspective is a rational demonstration, confirmed by experience, that all objects transmit their image to the eye by a pyramid of lines.

By a pyramid of lines I understand those lines which start from the edges of the surface of bodies, and converging from a distance, meet in a single point; and this point, in the present instance, I will show to be situated in the eye which is the universal judge of all objects. By a point I mean that which cannot be divided into parts; therefore this point, which is situated in the eye, being indivisible, no body is seen by the eye, that is not larger than this point. This being the case it is inevitable that the lines which come from the object to the point must form a pyramid. And if any man seeks to prove that the sense of sight does not reside in this point, but rather in the black spot which is visible in the middle of the pupil, I might reply to him that a small object could never diminish at any distance, as it might be a grain of millet or of oats or of some similar thing, and that object, if it were larger

than the said [black] spot would never be seen as a whole; as may be seen in the diagram below. Let a. be the seat of sight, b e the lines which reach the eye. Let e d be the grains of millet within these lines. You plainly see that these will never diminish by distance, and that the body m n could not be entirely covered by it. Therefore you must confess that the eye contains within itself one single indivisible point a, to which all the points converge of the pyramid of lines starting from an object, as is shown below. Let a. b. be the eye; in the centre of it is the point above mentioned. If the line e f is to enter as an image into so small an opening in the eye, you must confess that the smaller object cannot enter into what is smaller than itself unless it is diminished, and by diminishing it must take the form of a pyramid.

53.

PERSPECTIVE.

Perspective comes in where judgment fails [as to the distance] in objects which diminish. The eye can never be a true judge for determining with exactitude how near one object is to another which is equal to it [in size], if the top of that other is on the level of the eye which sees them on that side, excepting by means of the vertical plane which is the standard and guide of perspective. Let n be the eye, e f the vertical plane above mentioned. Let a b c d be the three divisions, one below the other; if the lines a n

and c n are of a given length and the eye n is in the centre, then a b will look as large as b c. c d is lower and farther off from n, therefore it will look smaller. And the same effect will appear in the three divisions of a face when the eye of the painter who is drawing it is on a level with the eye of the person he is painting.

54.

TO PROVE HOW OBJECTS REACH THE EYE.

If you look at the sun or some other luminous body and then shut your eyes you will see it again inside your eye for a long time. This is evidence that images enter into the eye.

The relations of the distance points to the vanishing point (55-56).

55.

ELEMENTS OF PERSPECTIVE.

All objects transmit their image to the eye in pyramids, and the nearer to the eye these pyramids are intersected the smaller will the image appear of the objects which cause them. Therefore, you may intersect the pyramid with a vertical plane [Footnote 4: Pariete. Compare the definitions in 85, 2-5, 6-27. These lines refer

exclusively to the third diagram. For the better understanding of this it should be observed that c s must be regarded as representing the section or profile of a square plane, placed horizontally (comp. lines 11, 14, 17) for which the word pianura is subsequently employed (20, 22). Lines 6-13 contain certain preliminary observations to guide the reader in understanding the diagram; the last three seem to have been added as a supplement. Leonardo's mistake in writing t denota (line 6) for f denota has been rectified.] which reaches the base of the pyramid as is shown in the plane a n.

The eye f and the eye t are one and the same thing; but the eye f marks the distance, that is to say how far you are standing from the object; and the eye t shows you the direction of it; that is whether you are opposite, or on one side, or at an angle to the object you are looking at. And remember that the eye f and the eye t must always be kept on the same level. For example if you raise or lower the eye from the distance point f you must do the same with the direction point t. And if the point f shows how far the eye is distant from the square plane but does not show on which side it is placed--and, if in the same way, the point t show s the direction and not the distance, in order to ascertain both you must use both points and they will be one and the same thing. If the eye f could see a perfect square of which all the sides were equal to the distance between s and c, and if at the nearest end of the side towards the eye a pole were placed, or some other straight

object, set up by a perpendicular line as shown at r s--then, I say, that if you were to look at the side of the square that is nearest to you it will appear at the bottom of the vertical plane r s, and then look at the farther side and it would appear to you at the height of the point n on the vertical plane. Thus, by this example, you can understand that if the eye is above a number of objects all placed on the same level, one beyond another, the more remote they are the higher they will seem, up to the level of the eye, but no higher; because objects placed upon the level on which your feet stand, so long as it is flat--even if it be extended into infinity--would never be seen above the eye; since the eye has in itself the point towards which all the cones tend and converge which convey the images of the objects to the eye. And this point always coincides with the point of diminution which is the extreme of all we can see. And from the base line of the first pyramid as far as the diminishing point

[Footnote: The two diagrams above the chapter are explained by the first five lines. They have, however, more letters than are referred to in the text, a circumstance we frequently find occasion to remark.]

56.

there are only bases without pyramids which constantly diminish up to this point. And from the first base where the vertical plane is

placed towards the point in the eye there will be only pyramids without bases; as shown in the example given above. Now, let a b be the said vertical plane and r the point of the pyramid terminating in the eye, and n the point of diminution which is always in a straight line opposite the eye and always moves as the eye moves--just as when a rod is moved its shadow moves, and moves with it, precisely as the shadow moves with a body. And each point is the apex of a pyramid, all having a common base with the intervening vertical plane. But although their bases are equal their angles are not equal, because the diminishing point is the termination of a smaller angle than that of the eye. If you ask me: "By what practical experience can you show me these points?" I reply--so far as concerns the diminishing point which moves with you --when you walk by a ploughed field look at the straight furrows which come down with their ends to the path where you are walking, and you will see that each pair of furrows will look as though they tried to get nearer and meet at the [farther] end.

[Footnote: For the easier understanding of the diagram and of its connection with the preceding I may here remark that the square plane shown above in profile by the line c s is here indicated by e d o p. According to lines 1, 3 a b must be imagined as a plane of glass placed perpendicularly at o p.]

57.

How to measure the pyramid of vision.

As regards the point in the eye; it is made more intelligible by this: If you look into the eye of another person you will see your own image. Now imagine 2 lines starting from your ears and going to the ears of that image which you see in the other man's eye; you will understand that these lines converge in such a way that they would meet in a point a little way beyond your own image mirrored in the eye. And if you want to measure the diminution of the pyramid in the air which occupies the space between the object seen and the eye, you must do it according to the diagram figured below. Let m n be a tower, and e f a, rod, which you must move backwards and forwards till its ends correspond with those of the tower [Footnote 9: I sua stremi .. della storre (its ends ... of the tower) this is the case at e f.]; then bring it nearer to the eye, at c d and you will see that the image of the tower seems smaller, as at r o. Then [again] bring it closer to the eye and you will see the rod project far beyond the image of the tower from a to b and from t to b, and so you will discern that, a little farther within, the lines must converge in a point.

The Production of pyramid of Vision (58-60).

58.

PERSPECTIVE.

The instant the atmosphere is illuminated it will be filled with an infinite number of images which are produced by the various bodies and colours assembled in it. And the eye is the target, a loadstone, of these images.

59.

The whole surface of opaque bodies displays its whole image in all the illuminated atmosphere which surrounds them on all sides.

60.

That the atmosphere attracts to itself, like a loadstone, all the images of the objects that exist in it, and not their forms merely but their nature may be clearly seen by the sun, which is a hot and luminous body. All the atmosphere, which is the all-pervading matter, absorbs light and heat, and reflects in itself the image of the source of that heat and splendour and, in each minutest portion, does the same. The Northpole does the same as the loadstone shows; and the moon and the other planets, without suffering any diminution, do the same. Among terrestrial things musk does the same and other perfumes.

61.

All bodies together, and each by itself, give off to the surrounding air an infinite number of images which are all-pervading and each complete, each conveying the nature, colour and form of the body which produces it.

It can clearly be shown that all bodies are, by their images, all-pervading in the surrounding atmosphere, and each complete in itself as to substance form and colour; this is seen by the images of the various bodies which are reproduced in one single perforation through which they transmit the objects by lines which intersect and cause reversed pyramids, from the objects, so that they are upside down on the dark plane where they are first reflected. The reason of this is--

[Footnote: The diagram intended to illustrate the statement (Pl. II No. i) occurs in the original between lines 3 and 4. The three circles must be understood to represent three luminous bodies which transmit their images through perforations in a wall into a dark chamber, according to a law which is more fully explained in 75?81. So far as concerns the present passage the diagram is only intended to explain that the images of the three bodies may be made to coalesce at any given spot. In the circles are written, giallo--yellow, biàcho--white, rosso--red.

The text breaks off at line 8. The paragraph No.40 follows here in the original MS.]

62.

Every point is the termination of an infinite number of lines, which diverge to form a base, and immediately, from the base the same lines converge to a pyramid [imaging] both the colour and form. No sooner is a form created or compounded than suddenly infinite lines and angles are produced from it; and these lines, distributing themselves and intersecting each other in the air, give rise to an infinite number of angles opposite to each other. Given a base, each opposite angle, will form a triangle having a form and proportion equal to the larger angle; and if the base goes twice into each of the 2 lines of the pyramid the smaller triangle will do the same.

63.

Every body in light and shade fills the surrounding air with infinite images of itself; and these, by infinite pyramids diffused in the air, represent this body throughout space and on every side. Each pyramid that is composed of a long assemblage of rays includes within itself an infinite number of pyramids and each has the same power as all, and all as each. A circle of equidistant pyramids of vision will give to their object angles of equal size; and an eye at each point will see the object of the same size. The body of the atmosphere is full of infinite pyramids composed of radiating straight lines, which are produced from the surface of the bodies in

light and shade, existing in the air; and the farther they are from the object which produces them the more acute they become and although in their distribution they intersect and cross they never mingle together, but pass through all the surrounding air, independently converging, spreading, and diffused. And they are all of equal power [and value]; all equal to each, and each equal to all. By these the images of objects are transmitted through all space and in every direction, and each pyramid, in itself, includes, in each minutest part, the whole form of the body causing it.

64.

The body of the atmosphere is full of infinite radiating pyramids produced by the objects existing in it. These intersect and cross each other with independent convergence without interfering with each other and pass through all the surrounding atmosphere; and are of equal force and value--all being equal to each, each to all. And by means of these, images of the body are transmitted everywhere and on all sides, and each receives in itself every minutest portion of the object that produces it.

Proof by experiment (65-66).

65.

PERSPECTIVE.

The air is filled with endless images of the objects distributed in it; and all are represented in all, and all in one, and all in each, whence it happens that if two mirrors are placed in such a manner as to face each other exactly, the first will be reflected in the second and the second in the first. The first being reflected in the second takes to it the image of itself with all the images represented in it, among which is the image of the second mirror, and so, image within image, they go on to infinity in such a manner as that each mirror has within it a mirror, each smaller than the last and one inside the other. Thus, by this example, it is clearly proved that every object sends its image to every spot whence the object itself can be seen; and the converse: That the same object may receive in itself all the images of the objects that are in front of it. Hence the eye transmits through the atmosphere its own image to all the objects that are in front of it and receives them into itself, that is to say on its surface, whence they are taken in by the common sense, which considers them and if they are pleasing commits them to the memory. Whence I am of opinion: That the invisible images in the eyes are produced towards the object, as the image of the object to the eye. That the images of the objects must be disseminated through the air. An instance may be seen in several mirrors placed in a circle, which will reflect each other endlessly. When one has reached the other it is returned to the object that produced it, and thence--being diminished--it is returned again to the object and then comes back once more, and this happens

endlessly. If you put a light between two flat mirrors with a distance of 1 braccio between them you will see in each of them an infinite number of lights, one smaller than another, to the last. If at night you put a light between the walls of a room, all the parts of that wall will be tinted with the image of that light. And they will receive the light and the light will fall on them, mutually, that is to say, when there is no obstacle to interrupt the transmission of the images. This same example is seen in a greater degree in the distribution of the solar rays which all together, and each by itself, convey to the object the image of the body which causes it. That each body by itself alone fills with its images the atmosphere around it, and that the same air is able, at the same time, to receive the images of the endless other objects which are in it, this is clearly proved by these examples. And every object is everywhere visible in the whole of the atmosphere, and the whole in every smallest part of it; and all the objects in the whole, and all in each smallest part; each in all and all in every part.

66.

The images of objects are all diffused through the atmosphere which receives them; and all on every side in it. To prove this, let a c e be objects of which the images are admitted to a dark chamber by the small holes n p and thrown upon the plane f i opposite to these holes. As many images will be produced in the chamber on the plane as the number of the said holes.

67.

General conclusions.

All objects project their whole image and likeness, diffused and mingled in the whole of the atmosphere, opposite to themselves. The image of every point of the bodily surface, exists in every part of the atmosphere. All the images of the objects are in every part of the atmosphere. The whole, and each part of the image of the atmosphere is [reflected] in each point of the surface of the bodies presented to it. Therefore both the part and the whole of the images of the objects exist, both in the whole and in the parts of the surface of these visible bodies. Whence we may evidently say that the image of each object exists, as a whole and in every part, in each part and in the whole interchangeably in every existing body. As is seen in two mirrors placed opposite to each other.

68.

That the contrary is impossible.

It is impossible that the eye should project from itself, by visual rays, the visual virtue, since, as soon as it opens, that front portion [of the eye] which would give rise to this emanation would have to go forth to the object and this it could not do without

time. And this being so, it could not travel so high as the sun in a month's time when the eye wanted to see it. And if it could reach the sun it would necessarily follow that it should perpetually remain in a continuous line from the eye to the sun and should always diverge in such a way as to form between the sun and the eye the base and the apex of a pyramid. This being the case, if the eye consisted of a million worlds, it would not prevent its being consumed in the projection of its virtue; and if this virtue would have to travel through the air as perfumes do, the winds would bent it and carry it into another place. But we do [in fact] see the mass of the sun with the same rapidity as [an object] at the distance of a braccio, and the power of sight is not disturbed by the blowing of the winds nor by any other accident.

[Footnote: The view here refuted by Leonardo was maintained among others by Bramantino, Leonardo's Milanese contemporary. LOMAZZO writes as follows in his Trattato dell' Arte della pittura &c. (Milano 1584. Libr. V cp. XXI): Sovviemmi di aver già letto in certi scritti alcune cose di Bramantino milanese, celebratissimo pittore, attenente alla prospettiva, le quali ho voluto riferire, e quasi intessere in questo luogo, affinchè sappiamo qual fosse l'opinione di cosi chiaro e famoso pittore intorno alla prospettiva . . Scrive Bramantino che la prospettiva è una cosa che contrafà il naturale, e che ciò si fa in tre modi

Circa il primo modo che si fa con ragione, per essere la cosa in

poche parole conclusa da Bramantino in maniera che giudico non potersi dir meglio, contenendovi si tutta Parte del principio al fine, io riferirò per appunto le proprie parole sue (cp. XXII, Prima prospettiva di Bramantino). La prima prospettiva fa le cose di punto, e l'altra non mai, e la terza più appresso. Adunque la prima si dimanda prospettiva, cioè ragione, la quale fa l'effetto dell' occhio, facendo crescere e calare secondo gli effetti degli occhi. Questo crescere e calare non procede della cosa propria, che in se per esser lontana, ovvero vicina, per quello effetto non può crescere e sminuire, ma procede dagli effetti degli occhi, i quali sono piccioli, e perciò volendo vedere tanto gran cosa, bisogna che mandino fuora la virtù visiva, la quale si dilata in tanta larghezza, che piglia tutto quello che vuoi vedere, ed arrivando a quella cosa la vede dove è: e da lei agli occhi per quello circuito fino all' occhio, e tutto quello termine è pieno di quella cosa.

It is worthy of note that Leonardo had made his memorandum refuting this view, at Milan in 1492]

69.

A parallel case.

Just as a stone flung into the water becomes the centre and cause of many circles, and as sound diffuses itself in circles in the air: so any object, placed in the luminous atmosphere, diffuses itself in

circles, and fills the surrounding air with infinite images of itself. And is repeated, the whole every-where, and the whole in every smallest part. This can be proved by experiment, since if you shut a window that faces west and make a hole [Footnote: 6. Here the text breaks off.] . .

[Footnote: Compare LIBRI, Histoire des sciences mathématiques en Italie. Tome III, p. 43.]

The function of the eye as explained by the camera obscura (70. 71).

70.

If the object in front of the eye sends its image to the eye, the eye, on the other hand, sends its image to the object, and no portion whatever of the object is lost in the images it throws off, for any reason either in the eye or the object. Therefore we may rather believe it to be the nature and potency of our luminous atmosphere which absorbs the images of the objects existing in it, than the nature of the objects, to send their images through the air. If the object opposite to the eye were to send its image to the eye, the eye would have to do the same to the object, whence it might seem that these images were an emanation. But, if so, it would be necessary [to admit] that every object became rapidly smaller; because each object appears by its images in the surrounding atmosphere. That is: the whole object in the whole atmosphere, and

in each part; and all the objects in the whole atmosphere and all of them in each part; speaking of that atmosphere which is able to contain in itself the straight and radiating lines of the images projected by the objects. From this it seems necessary to admit that it is in the nature of the atmosphere, which subsists between the objects, and which attracts the images of things to itself like a loadstone, being placed between them.

PROVE HOW ALL OBJECTS, PLACED IN ONE POSITION, ARE ALL EVERYWHERE AND ALL IN EACH PART.

I say that if the front of a building--or any open piazza or field--which is illuminated by the sun has a dwelling opposite to it, and if, in the front which does not face the sun, you make a small round hole, all the illuminated objects will project their images through that hole and be visible inside the dwelling on the opposite wall which may be made white; and there, in fact, they will be upside down, and if you make similar openings in several places in the same wall you will have the same result from each. Hence the images of the illuminated objects are all everywhere on this wall and all in each minutest part of it. The reason, as we clearly know, is that this hole must admit some light to the said dwelling, and the light admitted by it is derived from one or many luminous bodies. If these bodies are of various colours and shapes the rays forming the images are of various colours and shapes, and so will

the representations be on the wall.

[Footnote: 70. 15--23. This section has already been published in the "Saggio delle Opere di Leonardo da Vinci" Milan 1872, pp. 13, 14. G. Govi observes upon it, that Leonardo is not to be regarded as the inventor of the Camera obscura, but that he was the first to explain by it the structure of the eye. An account of the Camera obscura first occurs in CESARE CESARINI's Italian version of Vitruvius, pub. 1523, four years after Leonardo's death. Cesarini expressly names Benedettino Don Papnutio as the inventor of the Camera obscura. In his explanation of the function of the eye by a comparison with the Camera obscura Leonardo was the precursor of G. CARDANO, Professor of Medicine at Bologna (died 1576) and it appears highly probable that this is, in fact, the very discovery which Leonardo ascribes to himself in section 21 without giving any further details.]

71.

HOW THE IMAGES OF OBJECTS RECEIVED BY THE EYE INTERSECT WITHIN THE

CRYSTALLINE HUMOUR OF THE EYE.

An experiment, showing how objects transmit their images or pictures, intersecting within the eye in the crystalline humour, is seen when by some small round hole penetrate the images of illuminated objects into a very dark chamber. Then, receive these

images on a white paper placed within this dark room and rather near to the hole and you will see all the objects on the paper in their proper forms and colours, but much smaller; and they will be upside down by reason of that very intersection. These images being transmitted from a place illuminated by the sun will seem actually painted on this paper which must be extremely thin and looked at from behind. And let the little perforation be made in a very thin plate of iron. Let a b e d e be the object illuminated by the sun and o r the front of the dark chamber in which is the said hole at n m. Let s t be the sheet of paper intercepting the rays of the images of these objects upside down, because the rays being straight, a on the right hand becomes k on the left, and e on the left becomes f on the right; and the same takes place inside the pupil.

[Footnote: This chapter is already known through a translation into French by VENTURI. Compare his 'Essai sur les ouvrages physico-mathématiques de L. da Vinci avec des fragments tirés de ses Manuscrits, apportés de l'Italie. Lu a la premiere classe de l'Institut national des Sciences et Arts.' Paris, An V (1797).]

The practice of perspective (72. 73).

72.

In the practice of perspective the same rules apply to light and to

the eye.

73.

The object which is opposite to the pupil of the eye is seen by that pupil and that which is opposite to the eye is seen by the pupil.

Refraction of the rays falling upon the eye (74. 75)

74.

The lines sent forth by the image of an object to the eye do not reach the point within the eye in straight lines.

75.

If the judgment of the eye is situated within it, the straight lines of the images are refracted on its surface because they pass through the rarer to the denser medium. If, when you are under water, you look at objects in the air you will see them out of their true place; and the same with objects under water seen from the air.

The intersection of the rays (76-82).

76.

The inversion of the images.

All the images of objects which pass through a window [glass pane] from the free outer air to the air confined within walls, are seen on the opposite side; and an object which moves in the outer air from east to west will seem in its shadow, on the wall which is lighted by this confined air, to have an opposite motion.

77.

THE PRINCIPLE ON WHICH THE IMAGES OF BODIES PASS IN BETWEEN THE MARGINS OF THE OPENINGS BY WHICH THEY ENTER.

What difference is there in the way in which images pass through narrow openings and through large openings, or in those which pass by the sides of shaded bodies? By moving the edges of the opening through which the images are admitted, the images of immovable objects are made to move. And this happens, as is shown in the 9th which demonstrates: [Footnote 11: per la 9a che dicie. When Leonardo refers thus to a number it serves to indicate marginal diagrams; this can in some instances be distinctly proved. The ninth sketch on the page W. L. 145 b corresponds to the middle sketch of the three reproduced.] the images of any object are all everywhere, and all in each part of the surrounding air. It follows that if one of the edges of the hole by which the images are admitted to a dark chamber is moved it cuts off those rays of the image that were in

contact with it and gets nearer to other rays which previously were remote from it &c.

OF THE MOVEMENT OF THE EDGE AT THE RIGHT OR LEFT, OR THE UPPER, OR LOWER EDGE.

If you move the right side of the opening the image on the left will move [being that] of the object which entered on the right side of the opening; and the same result will happen with all the other sides of the opening. This can be proved by the 2nd of this which shows: all the rays which convey the images of objects through the air are straight lines. Hence, if the images of very large bodies have to pass through very small holes, and beyond these holes recover their large size, the lines must necessarily intersect.

[Footnote: 77. 2. In the first of the three diagrams Leonardo had drawn only one of the two margins, et m.]

78.

Necessity has provided that all the images of objects in front of the eye shall intersect in two places. One of these intersections is in the pupil, the other in the crystalline lens; and if this were not the case the eye could not see so great a number of objects as it does. This can be proved, since all the lines which intersect do

so in a point. Because nothing is seen of objects excepting their surface; and their edges are lines, in contradistinction to the definition of a surface. And each minute part of a line is equal to a point; for smallest is said of that than which nothing can be smaller, and this definition is equivalent to the definition of the point. Hence it is possible for the whole circumference of a circle to transmit its image to the point of intersection, as is shown in the 4th of this which shows: all the smallest parts of the images cross each other without interfering with each other. These demonstrations are to illustrate the eye. No image, even of the smallest object, enters the eye without being turned upside down; but as it penetrates into the crystalline lens it is once more reversed and thus the image is restored to the same position within the eye as that of the object outside the eye.

79.

OF THE CENTRAL LINE OF THE EYE.

Only one line of the image, of all those that reach the visual virtue, has no intersection; and this has no sensible dimensions because it is a mathematical line which originates from a mathematical point, which has no dimensions.

According to my adversary, necessity requires that the central line of every image that enters by small and narrow openings into a dark

chamber shall be turned upside down, together with the images of the bodies that surround it.

80.

AS TO WHETHER THE CENTRAL LINE OF THE IMAGE CAN BE INTERSECTED, OR

NOT, WITHIN THE OPENING.

It is impossible that the line should intersect itself; that is, that its right should cross over to its left side, and so, its left side become its right side. Because such an intersection demands two lines, one from each side; for there can be no motion from right to left or from left to right in itself without such extension and thickness as admit of such motion. And if there is extension it is no longer a line but a surface, and we are investigating the properties of a line, and not of a surface. And as the line, having no centre of thickness cannot be divided, we must conclude that the line can have no sides to intersect each other. This is proved by the movement of the line a f to a b and of the line e b to e f, which are the sides of the surface a f e b. But if you move the line a b and the line e f, with the frontends a e, to the spot c, you will have moved the opposite ends f b towards each other at the point d. And from the two lines you will have drawn the straight line c d which cuts the middle of the intersection of these two lines at the point n without any intersection. For, you

imagine these two lines as having breadth, it is evident that by this motion the first will entirely cover the other--being equal with it--without any intersection, in the position c d. And this is sufficient to prove our proposition.

81.

HOW THE INNUMERABLE RAYS FROM INNUMERABLE IMAGES CAN CONVERGE TO A

POINT.

Just as all lines can meet at a point without interfering with each other--being without breadth or thickness--in the same way all the images of surfaces can meet there; and as each given point faces the object opposite to it and each object faces an opposite point, the converging rays of the image can pass through the point and diverge again beyond it to reproduce and re-magnify the real size of that image. But their impressions will appear reversed--as is shown in the first, above; where it is said that every image intersects as it enters the narrow openings made in a very thin substance.

Read the marginal text on the other side.

In proportion as the opening is smaller than the shaded body, so much less will the images transmitted through this opening intersect each other. The sides of images which pass through openings into a

dark room intersect at a point which is nearer to the opening in proportion as the opening is narrower. To prove this let a b be an object in light and shade which sends not its shadow but the image of its darkened form through the opening d e which is as wide as this shaded body; and its sides a b, being straight lines (as has been proved) must intersect between the shaded object and the opening; but nearer to the opening in proportion as it is smaller than the object in shade. As is shown, on your right hand and your left hand, in the two diagrams a b c n m o where, the right opening d e, being equal in width to the shaded object a b, the intersection of the sides of the said shaded object occurs half way between the opening and the shaded object at the point c. But this cannot happen in the left hand figure, the opening o being much smaller than the shaded object n m.

It is impossible that the images of objects should be seen between the objects and the openings through which the images of these bodies are admitted; and this is plain, because where the atmosphere is illuminated these images are not formed visibly.

When the images are made double by mutually crossing each other they are invariably doubly as dark in tone. To prove this let d e h be such a doubling which although it is only seen within the space between the bodies in b and i this will not hinder its being seen from f g or from f m; being composed of the images a b i k which run together in d e h.

[Footnote: 81. On the original diagram at the beginning of this chapter Leonardo has written "azurro" (blue) where in the facsimile I have marked A, and "giallo" (yellow) where B stands.]

[Footnote: 15--23. These lines stand between the diagrams I and III.]

[Footnote: 24--53. These lines stand between the diagrams I and II.]

[Footnote: 54--97 are written along the left side of diagram I.]

82.

An experiment showing that though the pupil may not be moved from its position the objects seen by it may appear to move from their places.

If you look at an object at some distance from you and which is below the eye, and fix both your eyes upon it and with one hand firmly hold the upper lid open while with the other you push up the under lid--still keeping your eyes fixed on the object gazed at--you will see that object double; one [image] remaining steady, and the other moving in a contrary direction to the pressure of your finger on the lower eyelid. How false the opinion is of those who say that this happens because the pupil of the eye is displaced from its

position.

How the above mentioned facts prove that the pupil acts upside down in seeing.

[Footnote: 82. 14--17. The subject indicated by these two headings is fully discussed in the two chapters that follow them in the original; but it did not seem to me appropriate to include them here.]

Demostration of perspective by means of a vertical glass plane (83-85).

83.

OF THE PLANE OF GLASS.

Perspective is nothing else than seeing place [or objects] behind a plane of glass, quite transparent, on the surface of which the objects behind that glass are to be drawn. These can be traced in pyramids to the point in the eye, and these pyramids are intersected on the glass plane.

84.

Pictorial perspective can never make an object at the same distance,

look of the same size as it appears to the eye. You see that the apex of the pyramid f c d is as far from the object c d as the same point f is from the object a b; and yet c d, which is the base made by the painter's point, is smaller than a b which is the base of the lines from the objects converging in the eye and refracted at s t, the surface of the eye. This may be proved by experiment, by the lines of vision and then by the lines of the painter's plumbline by cutting the real lines of vision on one and the same plane and measuring on it one and the same object.

85.

PERSPECTIVE.

The vertical plane is a perpendicular line, imagined as in front of the central point where the apex of the pyramids converge. And this plane bears the same relation to this point as a plane of glass would, through which you might see the various objects and draw them on it. And the objects thus drawn would be smaller than the originals, in proportion as the distance between the glass and the eye was smaller than that between the glass and the objects.

PERSPECTIVE.

The different converging pyramids produced by the objects, will show, on the plane, the various sizes and remoteness of the objects

causing them.

PERSPECTIVE.

All those horizontal planes of which the extremes are met by perpendicular lines forming right angles, if they are of equal width the more they rise to the level of eye the less this is seen, and the more the eye is above them the more will their real width be seen.

PERSPECTIVE.

The farther a spherical body is from the eye the more you will see of it.

The angle of sight varies with the distance (86-88)

86.

A simple and natural method; showing how objects appear to the eye without any other medium.

The object that is nearest to the eye always seems larger than another of the same size at greater distance. The eye m, seeing the spaces o v x, hardly detects the difference between them, and the. reason of this is that it is close to them [Footnote 6: It is

quite inconceivable to me why M. RAVAISSON, in a note to his French translation of this simple passage should have remarked: Il est clair que c'est par erreur que Leonard a ècrit per esser visino au lieu de per non esser visino. (See his printed ed. of MS. A. p. 38.)]; but if these spaces are marked on the vertical plane n o the space o v will be seen at o r, and in the same way the space v x will appear at r q. And if you carry this out in any place where you can walk round, it will look out of proportion by reason of the great difference in the spaces o r and r q. And this proceeds from the eye being so much below [near] the plane that the plane is foreshortened. Hence, if you wanted to carry it out, you would have [to arrange] to see the perspective through a single hole which must be at the point m, or else you must go to a distance of at least 3 times the height of the object you see. The plane o p being always equally remote from the eye will reproduce the objects in a satisfactory way, so that they may be seen from place to place.

87.

How every large mass sends forth its images, which may diminish through infinity.

The images of any large mass being infinitely divisible may be infinitely diminished.

88.

Objects of equal size, situated in various places, will be seen by different pyramids which will each be smaller in proportion as the object is farther off.

89.

Perspective, in dealing with distances, makes use of two opposite pyramids, one of which has its apex in the eye and the base as distant as the horizon. The other has the base towards the eye and the apex on the horizon. Now, the first includes the [visible] universe, embracing all the mass of the objects that lie in front of the eye; as it might be a vast landscape seen through a very small opening; for the more remote the objects are from the eye, the greater number can be seen through the opening, and thus the pyramid is constructed with the base on the horizon and the apex in the eye, as has been said. The second pyramid is extended to a spot which is smaller in proportion as it is farther from the eye; and this second perspective [= pyramid] results from the first.

90.

SIMPLE PERSPECTIVE.

Simple perspective is that which is constructed by art on a vertical plane which is equally distant from the eye in every part. Complex

perspective is that which is constructed on a ground-plan in which none of the parts are equally distant from the eye.

91.

PERSPECTIVE.

No surface can be seen exactly as it is, if the eye that sees it is not equally remote from all its edges.

92.

WHY WHEN AN OBJECT IS PLACED CLOSE TO THE EYE ITS EDGES ARE INDISTINCT.

When an object opposite the eye is brought too close to it, its edges must become too confused to be distinguished; as it happens with objects close to a light, which cast a large and indistinct shadow, so is it with an eye which estimates objects opposite to it; in all cases of linear perspective, the eye acts in the same way as the light. And the reason is that the eye has one leading line (of vision) which dilates with distance and embraces with true discernment large objects at a distance as well as small ones that are close. But since the eye sends out a multitude of lines which surround this chief central one and since these which are farthest from the centre in this cone of lines are less able to discern with

accuracy, it follows that an object brought close to the eye is not at a due distance, but is too near for the central line to be able to discern the outlines of the object. So the edges fall within the lines of weaker discerning power, and these are to the function of the eye like dogs in the chase which can put up the game but cannot take it. Thus these cannot take in the objects, but induce the central line of sight to turn upon them, when they have put them up. Hence the objects which are seen with these lines of sight have confused outlines.

The relative size of objects with regard to their distance from the eye (93-98).

93.

PERSPECTIVE.

Small objects close at hand and large ones at a distance, being seen within equal angles, will appear of the same size.

94.

PERSPECTIVE.

There is no object so large but that at a great distance from the eye it does not appear smaller than a smaller object near.

95.

Among objects of equal size that which is most remote from the eye will look the smallest. [Footnote: This axiom, sufficiently clear in itself, is in the original illustrated by a very large diagram, constructed like that here reproduced under No. 108.

The same idea is repeated in C. A. I a; I a, stated as follows:
Infra le cose d'equal grandeza quella si dimostra di minor figura che sara più distante dall' ochio.--]

96.

Why an object is less distinct when brought near to the eye, and why with spectacles, or without the naked eye sees badly either close or far off [as the case may be].

97.

PERSPECTIVE.

Among objects of equal size, that which is most remote from the eye will look the smallest.

98.

PERSPECTIVE.

No second object can be so much lower than the first as that the eye will not see it higher than the first, if the eye is above the second.

PERSPECTIVE.

And this second object will never be so much higher than the first as that the eye, being below them, will not see the second as lower than the first.

PERSPECTIVE.

If the eye sees a second square through the centre of a smaller one, that is nearer, the second, larger square will appear to be surrounded by the smaller one.

PERSPECTIVE--PROPOSITION.

Objects that are farther off can never be so large but that those in front, though smaller, will conceal or surround them.

DEFINITION.

This proposition can be proved by experiment. For if you look through a small hole there is nothing so large that it cannot be seen through it and the object so seen appears surrounded and enclosed by the outline of the sides of the hole. And if you stop it up, this small stopping will conceal the view of the largest object.

The apparent size of objects defined by calculation (99-105)

99.

OF LINEAR PERSPECTIVE.

Linear Perspective deals with the action of the lines of sight, in proving by measurement how much smaller is a second object than the first, and how much the third is smaller than the second; and so on by degrees to the end of things visible. I find by experience that if a second object is as far beyond the first as the first is from the eye, although they are of the same size, the second will seem half the size of the first and if the third object is of the same size as the 2nd, and the 3rd is as far beyond the second as the 2nd from the first, it will appear of half the size of the second; and so on by degrees, at equal distances, the next farthest will be half the size of the former object. So long as the space does not exceed the length of 20 braccia. But, beyond 20 braccia figures of equal size will lose 2/4 and at 40 braccia they will lose 9/10, and 19/20 at 60 braccia, and so on diminishing by degrees. This is if the

picture plane is distant from you twice your own height. If it is only as far off as your own height, there will be a great difference between the first braccia and the second.

[Footnote: This chapter is included in DUFRESNE'S and MANZI'S editions of the Treatise on Painting. H. LUDWIG, in his commentary, calls this chapter "eines der wichtigsten im ganzen Tractat", but at the same time he asserts that its substance has been so completely disfigured in the best MS. copies that we ought not to regard Leonardo as responsible for it. However, in the case of this chapter, the old MS. copies agree with the original as it is reproduced above. From the chapters given later in this edition, which were written at a subsequent date, it would appear that Leonardo corrected himself on these points.]

100.

OF THE DIMINUTION OF OBJECTS AT VARIOUS DISTANCES.

A second object as far distant from the first as the first is from the eye will appear half the size of the first, though they be of the same size really.

OF THE DEGREES OF DIMINUTION.

If you place the vertical plane at one braccio from the eye, the

first object, being at a distance of 4 braccia from your eye will diminish to 3/4 of its height at that plane; and if it is 8 braccia from the eye, to 7/8; and if it is 16 braccia off, it will diminish to 15/16 of its height and so on by degrees, as the space doubles the diminution will double.

101.

Begin from the line m f with the eye below; then go up and do the same with the line n f, then with the eye above and close to the 2 gauges on the ground look at m n; then as c m is to m n so will n m be to n s.

If a n goes 3 times into f b, m p will do the same into p g.
Then go backwards so far as that c d goes twice into a n and p g will be equal to g h. And m p will go into h p as often as d c into o p.

[Footnote: The first three lines are unfortunately very obscure.]

102.

I GIVE THE DEGREES OF THE OBJECTS SEEN BY THE EYE AS THE MUSICIAN DOES THE NOTES HEARD BY THE EAR.

Although the objects seen by the eye do, in fact, touch each other

as they recede, I will nevertheless found my rule on spaces of 20 braccia each; as a musician does with notes, which, though they can be carried on one into the next, he divides into degrees from note to note calling them 1st, 2nd, 3rd, 4th, 5th; and has affixed a name to each degree in raising or lowering the voice.

103.

PERSPECTIVE.

Let f be the level and distance of the eye; and a the vertical plane, as high as a man; let e be a man, then I say that on the plane this will be the distance from the plane to the 2nd man.

104.

The differences in the diminution of objects of equal size in consequence of their various remoteness from the eye will bear among themselves the same proportions as those of the spaces between the eye and the different objects.

Find out how much a man diminishes at a certain distance and what its length is; and then at twice that distance and at 3 times, and so make your general rule.

105.

The eye cannot judge where an object high up ought to descend.

106.

PERSPECTIVE.

If two similar and equal objects are placed one beyond the other at a given distance the difference in their size will appear greater in proportion as they are nearer to the eye that sees them. And conversely there will seem to be less difference in their size in proportion as they are remote from the eve.

This is proved by the proportions of their distances among themselves; for, if the first of these two objects were as far from the eye, as the 2nd from the first this would be called the second proportion: since, if the first is at 1 braccia from the eye and the 2nd at two braccia, two being twice as much as one, the first object will look twice as large as the second. But if you place the first at a hundred braccia from you and the second at a hundred and one, you will find that the first is only so much larger than the second as 100 is less than 101; and the converse is equally true. And again, the same thing is proved by the 4th of this book which shows that among objects that are equal, there is the same proportion in the diminution of the size as in the increase in the distance from the eye of the spectator.

On natural perspective (107--109).

107.

OF EQUAL OBJECTS THE MOST REMOTE LOOK THE SMALLEST.

The practice of perspective may be divided into ... parts [Footnote 4: in ... parte. The space for the number is left blank in the original.], of which the first treats of objects seen by the eye at any distance; and it shows all these objects just as the eye sees them diminished, without obliging a man to stand in one place rather than another so long as the plane does not produce a second foreshortening.

But the second practice is a combination of perspective derived partly from art and partly from nature and the work done by its rules is in every portion of it, influenced by natural perspective and artificial perspective. By natural perspective I mean that the plane on which this perspective is represented is a flat surface, and this plane, although it is parallel both in length and height, is forced to diminish in its remoter parts more than in its nearer ones. And this is proved by the first of what has been said above, and its diminution is natural. But artificial perspective, that is that which is devised by art, does the contrary; for objects equal in size increase on the plane where it is foreshortened in

proportion as the eye is more natural and nearer to the plane, and as the part of the plane on which it is figured is farther from the eye.

And let this plane be d e on which are seen 3 equal circles which are beyond this plane d e, that is the circles a b c. Now you see that the eye h sees on the vertical plane the sections of the images, largest of those that are farthest and smallest of the nearest.

108.

Here follows what is wanting in the margin at the foot on the other side of this page.

Natural perspective acts in a contrary way; for, at greater distances the object seen appears smaller, and at a smaller distance the object appears larger. But this said invention requires the spectator to stand with his eye at a small hole and then, at that small hole, it will be very plain. But since many (men's) eyes endeavour at the same time to see one and the same picture produced by this artifice only one can see clearly the effect of this perspective and all the others will see confusion. It is well therefore to avoid such complex perspective and hold to simple perspective which does not regard planes as foreshortened, but as much as possible in their proper form. This simple perspective, in

which the plane intersects the pyramids by which the images are conveyed to the eye at an equal distance from the eye is our constant experience, from the curved form of the pupil of the eye on which the pyramids are intersected at an equal distance from the visual virtue.

[Footnote 24: la prima di sopra i. e. the first of the three diagrams which, in the original MS., are placed in the margin at the beginning of this chapter.]

109.

OF A MIXTURE OF NATURAL AND ARTIFICIAL PERSPECTIVE.

This diagram distinguishes natural from artificial perspective. But before proceeding any farther I will define what is natural and what is artificial perspective. Natural perspective says that the more remote of a series of objects of equal size will look the smaller, and conversely, the nearer will look the larger and the apparent size will diminish in proportion to the distance. But in artificial perspective when objects of unequal size are placed at various distances, the smallest is nearer to the eye than the largest and the greatest distance looks as though it were the least of all; and the cause of this is the plane on which the objects are represented; and which is at unequal distances from the eye throughout its length. And this diminution of the plane is natural, but the