CHAPTER IV.

A CHAPTER FOR THE CORNELL GIRLS.

No incident worth recording occurred during the night, if night indeed it could be called. In reality there was now no night or even day in the Projectile, or rather, strictly speaking, it was always night on the upper end of the bullet, and always day on the lower. Whenever, therefore, the words night and day occur in our story, the reader will readily understand them as referring to those spaces of time that are so called in our Earthly almanacs, and were so measured by the travellers' chronometers.

The repose of our friends must indeed have been undisturbed, if absolute freedom from sound or jar of any kind could secure tranquillity. In spite of its immense velocity, the Projectile still seemed to be perfectly motionless. Not the slightest sign of movement could be detected. Change of locality, though ever so rapid, can never reveal itself to our senses when it takes place in a vacuum, or when the enveloping atmosphere travels at the same rate as the moving body. Though we are incessantly whirled around the Sun at the rate of about seventy thousand miles an hour, which of us is conscious of the slightest motion? In such a case, as far as sensation is concerned, motion and repose are absolutely identical. Neither has any effect one way or another on a material body. Is such a body in motion? It remains in motion until some obstacle stops it. Is it at rest? It remains at

rest until some superior force compels it to change its position. This indifference of bodies to motion or rest is what physicists call inertia.

Barbican and his companions, therefore, shut up in the Projectile, could readily imagine themselves to be completely motionless. Had they been outside, the effect would have been precisely the same. No rush of air, no jarring sensation would betray the slightest movement. But for the sight of the Moon gradually growing larger above them, and of the Earth gradually growing smaller beneath them, they could safely swear that they were fast anchored in an ocean of deathlike immobility.

Towards the morning of next day (December 3), they were awakened by a joyful, but quite unexpected sound.

"Cock-a-doodle! doo!" accompanied by a decided flapping of wings.

The Frenchman, on his feet in one instant and on the top of the ladder in another, attempted to shut the lid of a half open box, speaking in an angry but suppressed voice:

"Stop this hullabaloo, won't you? Do you want me to fail in my great combination!"

"Hello?" cried Barbican and M'Nicholl, starting up and rubbing their eyes.

"What noise was that?" asked Barbican.

"Seems to me I heard the crowing of a cock," observed the Captain.

"I never thought your ears could be so easily deceived, Captain," cried Ardan, quickly, "Let us try it again," and, flapping his ribs with his arms, he gave vent to a crow so loud and natural that the lustiest chanticleer that ever saluted the orb of day might be proud of it.

The Captain roared right out, and even Barbican snickered, but as they saw that their companion evidently wanted to conceal something, they immediately assumed straight faces and pretended to think no more about the matter.

"Barbican," said Ardan, coming down the ladder and evidently anxious to change the conversation, "have you any idea of what I was thinking about all night?"

"Not the slightest."

"I was thinking of the promptness of the reply you received last year from the authorities of Cambridge University, when you asked them about the feasibility of sending a bullet to the Moon. You know very well by this time what a perfect ignoramus I am in Mathematics. I own I have been often puzzled when thinking on what grounds they could form such a positive opinion, in a case where I am certain that the calculation must be an exceedingly delicate matter."

"The feasibility, you mean to say," replied Barbican, "not exactly of sending a bullet to the Moon, but of sending it to the neutral point between the Earth and the Moon, which lies at about nine-tenths of the journey, where the two attractions counteract each other. Because that point once passed, the Projectile would reach the Moon's surface by virtue of its own weight."

"Well, reaching that neutral point be it;" replied Ardan, "but, once more, I should like to know how they have been able to come at the necessary initial velocity of 12,000 yards a second?"

"Nothing simpler," answered Barbican.

"Could you have done it yourself?" asked the Frenchman.

"Without the slightest difficulty. The Captain and myself could have readily solved the problem, only the reply from the University saved us the trouble."

"Well, Barbican, dear boy," observed Ardan, "all I've got to say is, you might chop the head off my body, beginning with my feet, before you could make me go through such a calculation."

"Simply because you don't understand Algebra," replied Barbican, quietly.

"Oh! that's all very well!" cried Ardan, with an ironical smile. "You great x+y men think you settle everything by uttering the word Algebra!"

"Ardan," asked Barbican, "do you think people could beat iron without a hammer, or turn up furrows without a plough?"

"Hardly."

"Well, Algebra is an instrument or utensil just as much as a hammer or a plough, and a very good instrument too if you know how to make use of it."

"You're in earnest?"

"Quite so."

"And you can handle the instrument right before my eyes?"

"Certainly, if it interests you so much."

"You can show me how they got at the initial velocity of our Projectile?"

"With the greatest pleasure. By taking into proper consideration all the elements of the problem, viz.: (1) the distance between the centres of the Earth and the Moon, (2) the Earth's radius, (3) its volume, and (4)

the Moon's volume, I can easily calculate what must be the initial velocity, and that too by a very simple formula."

"Let us have the formula."

"In one moment; only I can't give you the curve really described by the Projectile as it moves between the Earth and the Moon; this is to be obtained by allowing for their combined movement around the Sun. I will consider the Earth and the Sun to be motionless, that being sufficient for our present purpose."

"Why so?"

"Because to give you that exact curve would be to solve a point in the 'Problem of the Three Bodies,' which Integral Calculus has not yet reached."

"What!" cried Ardan, in a mocking tone, "is there really anything that Mathematics can't do?"

"Yes," said Barbican, "there is still a great deal that Mathematics can't even attempt."

"So far, so good;" resumed Ardan. "Now then what is this Integral Calculus of yours?"

"It is a branch of Mathematics that has for its object the summation of

a certain infinite series of indefinitely small terms: but for the solution of which, we must generally know the function of which a given function is the differential coefficient. In other words," continued Barbican, "in it we return from the differential coefficient, to the function from which it was deduced."

"Clear as mud!" cried Ardan, with a hearty laugh.

"Now then, let me have a bit of paper and a pencil," added Barbican, "and in half an hour you shall have your formula; meantime you can easily find something interesting to do."

In a few seconds Barbican was profoundly absorbed in his problem, while M'Nicholl was watching out of the window, and Ardan was busily employed in preparing breakfast.

The morning meal was not quite ready, when Barbican, raising his head, showed Ardan a page covered with algebraic signs at the end of which stood the following formula:--

$$\frac{1}{2}(v' - v)^2 = gr\left\{\frac{r}{x} - 1 + \frac{m'}{m}\left(\frac{r}{d-x} - \frac{r}{d-r}\right)\right\}$$

"Which means?" asked Ardan.

"It means," said the Captain, now taking part in the discussion, "that

the half of v prime squared minus v squared equals gr multiplied by r over x minus one plus m prime over m multiplied by r over d minus x minus r over d minus r ... that is--"

"That is," interrupted Ardan, in a roar of laughter, "x stradlegs on y, making for z and jumping over p! Do you mean to say you understand the terrible jargon, Captain?"

"Nothing is clearer, Ardan."

"You too, Captain! Then of course I must give in gracefully, and declare that the sun at noon-day is not more palpably evident than the sense of Barbican's formula."

"You asked for Algebra, you know," observed Barbican.

"Rock crystal is nothing to it!"

"The fact is, Barbican," said the Captain, who had been looking over the paper, "you have worked the thing out very well. You have the integral equation of the living forces, and I have no doubt it will give us the result sought for."

"Yes, but I should like to understand it, you know," cried Ardan: "I would give ten years of the Captain's life to understand it!"

"Listen then," said Barbican. "Half of v prime squared less v

squared, is the formula giving us the half variation of the living force."

"Mac pretends he understands all that!"

"You need not be a Solomon to do it," said the Captain. "All these signs that you appear to consider so cabalistic form a language the clearest, the shortest, and the most logical, for all those who can read it."

"You pretend, Captain, that, by means of these hieroglyphics, far more incomprehensible than the sacred Ibis of the Egyptians, you can discover the velocity at which the Projectile should start?"

"Most undoubtedly," replied the Captain, "and, by the same formula I can even tell you the rate of our velocity at any particular point of our journey."

"You can?"

"I can."

"Then you're just as deep a one as our President."

"No, Ardan; not at all. The really difficult part of the question Barbican has done. That is, to make out such an equation as takes into account all the conditions of the problem. After that, it's a simple

affair of Arithmetic, requiring only a knowledge of the four rules to work it out."

"Very simple," observed Ardan, who always got muddled at any kind of a difficult sum in addition.

"Captain," said Barbican, "you could have found the formulas too, if you tried."

"I don't know about that," was the Captain's reply, "but I do know that this formula is wonderfully come at."

"Now, Ardan, listen a moment," said Barbican, "and you will see what sense there is in all these letters."

"I listen," sighed Ardan with the resignation of a martyr.

"d is the distance from the centre of the Earth to the centre of the Moon, for it is from the centres that we must calculate the attractions."

"That I comprehend."

"r is the radius of the Earth."

"That I comprehend."

"m is the mass or volume of the Earth; m prime that of the Moon. We must take the mass of the two attracting bodies into consideration, since attraction is in direct proportion to their masses."

"That I comprehend."

"g is the gravity or the velocity acquired at the end of a second by a body falling towards the centre of the Earth. Clear?"

"That I comprehend."

"Now I represent by x the varying distance that separates the Projectile from the centre of the Earth, and by v prime its velocity at that distance."

"That I comprehend."

"Finally, v is its velocity when quitting our atmosphere."

"Yes," chimed in the Captain, "it is for this point, you see, that the velocity had to be calculated, because we know already that the initial velocity is exactly the three halves of the velocity when the Projectile quits the atmosphere."

"That I don't comprehend," cried the Frenchman, energetically.

"It's simple enough, however," said Barbican.

"Not so simple as a simpleton," replied the Frenchman.

"The Captain merely means," said Barbican, "that at the instant the Projectile quitted the terrestrial atmosphere it had already lost a third of its initial velocity."

"So much as a third?"

"Yes, by friction against the atmospheric layers: the quicker its motion, the greater resistance it encountered."

"That of course I admit, but your v squared and your v prime squared rattle in my head like nails in a box!"

"The usual effect of Algebra on one who is a stranger to it; to finish you, our next step is to express numerically the value of these several symbols. Now some of them are already known, and some are to be calculated."

"Hand the latter over to me," said the Captain.

"First," continued Barbican: "r, the Earth's radius is, in the latitude of Florida, about 3,921 miles. d, the distance from the centre of the Earth to the centre of the Moon is 56 terrestrial radii, which the Captain calculates to be...?"

"To be," cried M'Nicholl working rapidly with his pencil, "219,572 miles, the moment the Moon is in her perigee, or nearest point to the Earth."

"Very well," continued Barbican. "Now m prime over m, that is the ratio of the Moon's mass to that of the Earth is about the 1/81. g gravity being at Florida about 32-1/4 feet, of course g x r must be--how much, Captain?"

"38,465 miles," replied M'Nicholl.

"Now then?" asked Ardan.

"Now then," replied Barbican, "the expression having numerical values, I am trying to find v, that is to say, the initial velocity which the Projectile must possess in order to reach the point where the two attractions neutralize each other. Here the velocity being null, v prime becomes zero, and x the required distance of this neutral point must be represented by the nine-tenths of d, the distance between the two centres."

"I have a vague kind of idea that it must be so," said Ardan.

"I shall, therefore, have the following result;" continued Barbican, figuring up; "x being nine-tenths of d, and v prime being zero, my formula becomes:--

$$v^2 = gr\left\{1 - \frac{10r}{d} - \frac{1}{81}\left(\frac{10r}{d}\right)^2 - \left(\frac{r}{d-r}\right)^2\right\}$$

The Captain read it off rapidly.

"Right! that's correct!" he cried.

"You think so?" asked Barbican.

"As true as Euclid!" exclaimed M'Nicholl.

"Wonderful fellows," murmured the Frenchman, smiling with admiration.

"You understand now, Ardan, don't you?" asked Barbican.

"Don't I though?" exclaimed Ardan, "why my head is splitting with it!"

"Therefore," continued Barbican,

$$2v'^2 = 2gr\left\{1 - \frac{10r}{d} - \frac{1}{81}\left(\frac{10r}{d}\right)^2 - \left(\frac{r}{d-r}\right)^2\right\}$$

"And now," exclaimed M'Nicholl, sharpening his pencil; "in order to obtain the velocity of the Projectile when leaving the atmosphere, we have only to make a slight calculation."

The Captain, who before clerking on a Mississippi steamboat had been professor of Mathematics in an Indiana university, felt quite at home at the work. He rained figures from his pencil with a velocity that would have made Marston stare. Page after page was filled with his multiplications and divisions, while Barbican looked quietly on, and Ardan impatiently stroked his head and ears to keep down a rising head-ache.

"Well?" at last asked Barbican, seeing the Captain stop and throw a somewhat hasty glance over his work.

"Well," answered M'Nicholl slowly but confidently, "the calculation is made, I think correctly; and v, that is, the velocity of the Projectile when quitting the atmosphere, sufficient to carry it to the neutral point, should be at least ..."

"How much?" asked Barbican, eagerly.

"Should be at least 11,972 yards the first second."

"What!" cried Barbican, jumping off his seat. "How much did you say?"

"11,972 yards the first second it quits the atmosphere."

"Oh, malediction!" cried Barbican, with a gesture of terrible despair.

"What's the matter?" asked Ardan, very much surprised.

"Enough is the matter!" answered Barbican excitedly. "This velocity having been diminished by a third, our initial velocity should have been at least ..."

"17,958 yards the first second!" cried M'Nicholl, rapidly flourishing his pencil.

"But the Cambridge Observatory having declared that 12,000 yards the first second were sufficient, our Projectile started with no greater velocity!"

"Well?" asked M'Nicholl.

"Well, such a velocity will never do!"

"How??" }
"How!!" } cried the Captain and Ardan in one voice.

"We can never reach the neutral point!"

"Thunder and lightning"

"Fire and Fury!"

"We can't get even halfway!"

"Heaven and Earth!"

"Mille noms d'un boulet!" cried Ardan, wildly gesticulating.

"And we shall fall back to the Earth!"

"Oh!"

"Ah!"

They could say no more. This fearful revelation took them like a stroke of apoplexy.