CHAPTER IV.

A LITTLE ALGEBRA.

The night passed without incident. Correctly speaking, the word "night" is an improper one. The position of the projectile in regard to the sun did not change. Astronomically it was day on the bottom of the bullet, and night on the top. When, therefore, in this recital these two words are used they express the lapse of time between the rising and setting of the sun upon earth.

The travellers' sleep was so much the more peaceful because, notwithstanding its excessive speed, the projectile seemed absolutely motionless. No movement indicated its journey through space. However rapidly change of place may be effected, it cannot produce any sensible effect upon the organism when it takes place in the void, or when the mass of air circulates along with the travelling body. What inhabitant of the earth perceives the speed which carries him along at the rate of 68,000 miles an hour? Movement under such circumstances is not felt more than repose. Every object is indifferent to it. When a body is in repose it remains so until some foreign force puts it in movement. When in movement it would never stop if some obstacle were not in its road. This indifference to movement or repose is inertia.

Barbicane and his companions could, therefore, imagine themselves

absolutely motionless, shut up in the interior of the projectile. The effect would have been the same if they had placed themselves on the outside. Without the moon, which grew larger above them, and the earth that grew smaller below, they would have sworn they were suspended in a complete stagnation.

That morning, the 3rd of December, they were awakened by a joyful but unexpected noise. It was the crowing of a cock in the interior of their vehicle.

Michel Ardan was the first to get up; he climbed to the top of the projectile and closed a partly-open case.

"Be quiet," said he in a whisper. "That animal will spoil my plan!"

In the meantime Nicholl and Barbicane awoke.

"Was that a cock?" said Nicholl.

"No, my friends," answered Michel quickly. "I wished to awake you with that rural sound."

So saying he gave vent to a cock-a-doodle-do which would have done honour to the proudest of gallinaceans.

The two Americans could not help laughing.

"A fine accomplishment that," said Nicholl, looking suspiciously at his companion.

"Yes," answered Michel, "a joke common in my country. It is very Gallic. We perpetrate it in the best society."

Then turning the conversation--

"Barbicane, do you know what I have been thinking about all night?"

"No," answered the president.

"About our friends at Cambridge. You have already remarked how admirably ignorant I am of mathematics. I find it, therefore, impossible to guess how our savants of the observatory could calculate what initial velocity the projectile ought to be endowed with on leaving the Columbiad in order to reach the moon."

"You mean," replied Barbicane, "in order to reach that neutral point where the terrestrial and lunar attractions are equal; for beyond this point, situated at about 0.9 of the distance, the projectile will fall upon the moon by virtue of its own weight merely."

"Very well," answered Michel; "but once more; how did they calculate the initial velocity?"

"Nothing is easier," said Barbicane.

"And could you have made the calculation yourself?" asked Michel Ardan.

"Certainly; Nicholl and I could have determined it if the notice from the observatory had not saved us the trouble."

"Well, old fellow," answered Michel, "they might sooner cut off my head, beginning with my feet, than have made me solve that problem!"

"Because you do not know algebra," replied Barbicane tranquilly.

"Ah, that's just like you dealers in x! You think you have explained everything when you have said 'algebra.'"

"Michel," replied Barbicane, "do you think it possible to forge without a hammer, or to plough without a ploughshare?"

"It would be difficult."

"Well, then, algebra is a tool like a plough or a hammer, and a good tool for any one who knows how to use it."

"Seriously?"

"Quite."

"Could you use that tool before me?"

"If it would interest you."

"And could you show me how they calculated the initial speed of our vehicle?"

"Yes, my worthy friend. By taking into account all the elements of the problem, the distance from the centre of the earth to the centre of the moon, of the radius of the earth, the volume of the earth and the volume of the moon, I can determine exactly what the initial speed of the projectile ought to be, and that by a very simple formula."

"Show me the formula."

"You shall see it. Only I will not give you the curve really traced by the bullet between the earth and the moon, by taking into account their movement of translation round the sun. No. I will consider both bodies to be motionless, and that will be sufficient for us."

"Why?"

"Because that would be seeking to solve the problem called 'the problem of the three bodies,' for which the integral calculus is not yet far

enough advanced."

"Indeed," said Michel Ardan in a bantering tone; "then mathematics have not said their last word."

"Certainly not," answered Barbicane.

"Good! Perhaps the Selenites have pushed the integral calculus further than you! By-the-bye, what is the integral calculus?"

"It is the inverse of the differential calculus," answered Barbicane seriously.

"Much obliged."

"To speak otherwise, it is a calculus by which you seek finished quantities of what you know the differential quantities."

"That is clear at least," answered Barbicane with a quite satisfied air.

"And now," continued Barbicane, "for a piece of paper and a pencil, and in half-an-hour I will have found the required formula."

That said, Barbicane became absorbed in his work, whilst Nicholl looked into space, leaving the care of preparing breakfast to his companion.

Half-an-hour had not elapsed before Barbicane, raising his head, showed Michel Ardan a page covered with algebraical signs, amidst which the following general formula was discernible:--

$$\frac{1}{2}(v_2^2 - v_0^2) = gr\left\{\frac{x}{m}\left(1 + \frac{m'}{d-x}\right)\left(\frac{r}{d-r}\right)\right\}$$

"And what does that mean?" asked Michel.

"That means," answered Nicholl, "that the half of v minus v zero square equals gr multiplied by r upon x minus 1 plus m prime upon m multiplied by r upon d minus x, minus r upon d minus x minus r--"

"X upon y galloping upon z and rearing upon p" cried Michel Ardan, bursting out laughing. "Do you mean to say you understand that, captain?"

"Nothing is clearer."

"Then," said Michel Ardan, "it is as plain as a pikestaff, and I want nothing more."

"Everlasting laugher," said Barbicane, "you wanted algebra, and now you shall have it over head and ears."

"I would rather be hung!"

"That appears a good solution, Barbicane," said Nicholl, who was examining the formula like a connaisseur. "It is the integral of the equation of 'vis viva,' and I do not doubt that it will give us the desired result."

"But I should like to understand!" exclaimed Michel. "I would give ten years of Nicholl's life to understand!"

"Then listen," resumed Barbicane. "The half of v minus v zero square is the formula that gives us the demi-variation of the 'vis viva.'"

"Good; and does Nicholl understand what that means?"

"Certainly, Michel," answered the captain. "All those signs that look so cabalistic to you form the clearest and most logical language for those who know how to read it."

"And do you pretend, Nicholl," asked Michel, "that by means of these hieroglyphics, more incomprehensible than the Egyptian ibis, you can find the initial speed necessary to give to the projectile?"

"Incontestably," answered Nicholl; "and even by that formula I could always tell you what speed it is going at on any point of the journey."

"Upon your word of honour?"

"Yes."

"Then you are as clever as our president."

"No, Michel, all the difficulty consists in what Barbicane has done. It is to establish an equation which takes into account all the conditions of the problem. The rest is only a question of arithmetic, and requires nothing but a knowledge of the four rules."

"That's something," answered Michel Ardan, who had never been able to make a correct addition in his life, and who thus defined the rule: "A Chinese puzzle, by which you can obtain infinitely various results."

Still Barbicane answered that Nicholl would certainly have found the formula had he thought about it.

"I do not know if I should," said Nicholl, "for the more I study it the more marvellously correct I find it."

"Now listen," said Barbicane to his ignorant comrade, "and you will see that all these letters have a signification."

"I am listening," said Michel, looking resigned.

"d," said Barbicane, "is the distance from the centre of the earth to the centre of the moon, for we must take the centres to calculate the attraction."

"That I understand."

"r is the radius of the earth."

"r, radius; admitted."

"m is the volume of the earth; m prime that of the moon. We are obliged to take into account the volume of the two attracting bodies, as the attraction is in proportion to the volume."

"I understand that."

"g represents gravity, the speed acquired at the end of a second by a body falling on the surface of the earth. Is that clear?"

"A mountain stream!" answered Michel.

"Now I represent by x the variable distance that separates the projectile from the centre of the earth, and by v the velocity the projectile has at that distance."

"Good."

"Lastly, the expression v zero which figures in the equation is the speed the bullet possesses when it emerges from the atmosphere."

"Yes," said Nicholl, "you were obliged to calculate the velocity from that point, because we knew before that the velocity at departure is exactly equal to 3/2 of the velocity upon emerging from the atmosphere."

"Don't understand any more!" said Michel.

"Yet it is very simple," said Barbicane.

"I do not find it very simple," replied Michel.

"It means that when our projectile reached the limit of the terrestrial atmosphere it had already lost one-third of its initial velocity."

"As much as that?"

"Yes, my friend, simply by friction against the atmosphere. You will easily understand that the greater its speed the more resistance it would meet with from the air."

"That I admit," answered Michel, "and I understand it, although your v zero two and your v zero square shake about in my head like nails in a

sack."

"First effect of algebra," continued Barbicane. "And now to finish we are going to find the numerical known quantity of these different expressions--that is to say, find out their value."

"You will finish me first!" answered Michel.

"Some of these expressions," said Barbicane, "are known; the others have to be calculated."

"I will calculate those," said Nicholl.

"And r," resumed Barbicane, "r is the radius of the earth under the latitude of Florida, our point of departure, d--that is to say, the distance from the centre of the earth to the centre of the moon equals fifty-six terrestrial radii--"

Nicholl rapidly calculated.

"That makes 356,720,000 metres when the moon is at her perigee--that is to say, when she is nearest to the earth."

"Very well," said Barbicane, "now m prime upon m--that is to say, the proportion of the moon's volume to that of the earth equals 1/81."

"Perfect," said Michel.

"And g, the gravity, is to Florida 9-1/81 metres. From whence it results that gr equals--"

"Sixty-two million four hundred and twenty-six thousand square metres," answered Nicholl.

"What next?" asked Michel Ardan.

"Now that the expressions are reduced to figures, I am going to find the velocity v zero--that is to say, the velocity that the projectile ought to have on leaving the atmosphere to reach the point of equal attraction with no velocity. The velocity at that point I make equal zero, and x, the distance where the neutral point is, will be represented by the nine-tenths of d--that is to say, the distance that separates the two centres."

"I have some vague idea that it ought to be so," said Michel.

"I shall then have, x equals nine-tenths of d, and v equals zero, and my formula will become--"

Barbicane wrote rapidly on the paper--

$$2 \qquad 10r \quad 1 \quad 10r \quad r$$

$$v_0^2 = 2gr\left\{1 - \frac{10r}{9d} - \frac{1}{81}\cdot\frac{10r}{d}\left(1 - \frac{r}{d}\right)\right\}$$

Nicholl read it quickly.

"That's it! that is it!" he cried.

"Is it clear?" asked Barbicane.

"It is written in letters of fire!" answered Nicholl.

"Clever fellows!" murmured Michel.

"Do you understand now?" asked Barbicane.

"If I understand!" cried Michel Ardan. "My head is bursting with it."

"Thus," resumed Barbicane, "v zero square equals 2 gr multiplied by 1 minus 10 r upon 9 d minus 1/81 multiplied by 10 r upon d minus r upon d minus r."

"And now," said Nicholl, "in order to obtain the velocity of the bullet as it emerges from the atmosphere I have only to calculate."

The captain, like a man used to overcome all difficulties, began to calculate with frightful rapidity. Divisions and multiplications grew

under his fingers. Figures dotted the page. Barbicane followed him with his eyes, whilst Michel Ardan compressed a coming headache with his two hands.

"Well, what do you make it?" asked Barbicane after several minutes' silence.

"I make it 11,051 metres in the first second."

"What do you say?" said Barbicane, starting.

"Eleven thousand and fifty-one metres."

"Malediction!" cried the president with a gesture of despair.

"What's the matter with you?" asked Michel Ardan, much surprised.

"The matter! why if at this moment the velocity was already diminished one-third by friction, the initial speed ought to have been--"

"Sixteen thousand five hundred and seventy-six metres!" answered Nicholl.

"But the Cambridge Observatory declared that 11,000 metres were enough at departure, and our bullet started with that velocity only!"

"Well?" asked Nicholl.

"Why it was not enough!"

"No."

"We shall not reach the neutral point."

"The devil!"

"We shall not even go half way!"

"Nom d'un boulet!" exclaimed Michel Ardan, jumping up as if the projectile were on the point of striking against the terrestrial globe.

"And we shall fall back upon the earth!"