CHAPTER V.

OF DEMONSTRATION, AND NECESSARY TRUTHS.

§ 1. If, as laid down in the two preceding chapters, the foundation of all sciences, even deductive or demonstrative sciences, is Induction; if every step in the ratiocinations even of geometry is an act of induction; and if a train of reasoning is but bringing many inductions to bear upon the same subject of inquiry, and drawing a case within one induction by means of another; wherein lies the peculiar certainty always ascribed to the sciences which are entirely, or almost entirely, deductive? Why are they called the Exact Sciences? Why are mathematical certainty, and the evidence of demonstration, common phrases to express the very highest degree of assurance attainable by reason? Why are mathematics by almost all philosophers, and (by many) even those branches of natural philosophy which, through the medium of mathematics, have been converted into deductive sciences, considered to be independent of the evidence of experience and observation, and characterized as systems of Necessary Truth?

The answer I conceive to be, that this character of necessity, ascribed to the truths of mathematics, and even (with some reservations to be hereafter made) the peculiar certainty attributed to them, is an illusion; in order to sustain which, it is necessary to suppose that those truths relate to, and express the properties of, purely imaginary objects. It is acknowledged that the conclusions of geometry are deduced, partly at least, from the so-called Definitions, and that those definitions are assumed to be correct descriptions, as far as they go, of the objects with which geometry is conversant. Now we have pointed out that, from a definition as such, no proposition, unless it be one concerning the meaning of a word, can ever follow; and that what apparently follows from a definition, follows in reality from an implied assumption that there exists a real thing conformable thereto. This assumption, in the case of the definitions of geometry, is false: there exist no real things exactly conformable to the definitions. There exist no points without magnitude; no lines without breadth, nor perfectly straight; no circles with all their radii exactly equal, nor squares with all their angles perfectly right. It will perhaps be said that the assumption does not extend to the actual, but only to the possible, existence of such things. I answer that, according to any test we have of possibility, they are not even possible. Their existence, so far as we can form any judgment, would seem to be inconsistent with the physical constitution of our planet at least, if not of the universe. To get rid of this difficulty, and at the same time to save the credit of the supposed system of necessary truth, it is customary to say that the points, lines, circles, and squares which are the subject of geometry, exist in our conceptions merely, and are part of our minds; which minds, by working on their own materials, construct an *à priori* science, the evidence of which is purely mental, and has nothing whatever to do with outward experience. By howsoever high authorities this doctrine may have been sanctioned, it appears to me psychologically incorrect. The points, lines, circles, and squares, which any one has in his mind, are (I apprehend) simply copies of the points, lines, circles, and squares which he has known in his experience. Our idea of a point, I apprehend to be simply our idea of the *minimum visibile*, the smallest portion of surface which we can see. A line, as defined by geometers, is wholly inconceivable. We can reason about a line as if it had no breadth; because we have a power, which is the foundation of all the control we can exercise over the operations of our minds; the power, when a perception is present to our senses, or a conception to our intellects, of *attending* to a part only of that perception or conception, instead of the whole. But we cannot *conceive* a line without breadth; we can form no mental picture of such a line: all the lines which we have in our minds are lines possessing breadth. If any one doubts this, we may refer him to his own experience. I much question if any one who fancies that he can conceive what is called a mathematical line, thinks so from the evidence of his consciousness: I suspect it is rather because he supposes that unless such a conception were possible, mathematics could not exist as a science: a supposition which there will be no difficulty in showing to be entirely groundless.

Since, then, neither in nature, nor in the human mind, do there exist any objects exactly corresponding to the definitions of geometry, while yet that science cannot be supposed to be conversant about non-entities; nothing remains but to consider geometry as conversant with such lines, angles, and figures, as really exist; and the definitions, as they are called, must be regarded as some of our first and most obvious generalizations concerning those natural objects. The correctness of those generalizations, *as* generalizations, is without a flaw: the equality of all the radii of a circle is true of all circles, so far as it is true of any one: but it is not exactly true of any circle: it is only nearly true; so nearly that no error of any importance in practice will be incurred by feigning it to be exactly true. When we have occasion to extend these inductions, or their consequences, to cases in which the error would be appreciable--to lines of perceptible breadth or thickness, parallels which deviate sensibly from equidistance, and the like--we correct our conclusions, by combining with them a fresh set of propositions relating to the aberration; just as we also take in propositions relating to the physical or chemical properties of the material, if those properties happen to introduce any modification into the result; which they easily may, even with respect to figure and magnitude, as in the case, for instance, of expansion by heat. So long, however, as there exists no practical necessity for attending to any of the properties of the object except its geometrical properties, or to any of the natural irregularities in those, it is convenient to neglect the consideration of the other properties and of the irregularities, and to reason as if these did not exist: accordingly, we formally announce, in the definitions, that we intend to proceed on this plan. But it is an error to suppose, because we resolve to confine our attention to a certain number of the properties of an object, that we therefore conceive, or have an idea of the object, denuded of its other properties. We are thinking, all the time, of precisely such objects as we have seen and touched, and with all the properties which naturally belong to them; but for scientific convenience, we feign them to be divested of all properties, except those which are material to our purpose, and in regard to which we design to consider them.

The peculiar accuracy, supposed to be characteristic of the first principles of geometry, thus appears to be fictitious. The assertions on which the reasonings of the science are founded, do not, any more than in other sciences, exactly correspond with the fact; but we *suppose* that they do so, for the sake of tracing the consequences which follow from the supposition. The opinion of Dugald Stewart respecting the foundations of geometry, is, I conceive, substantially correct; that it is built on hypotheses; that it owes to this alone the peculiar certainty supposed to distinguish it; and that in any science whatever, by reasoning from a set of hypotheses, we may obtain a body of conclusions as certain as those of geometry, that is, as strictly in accordance with the hypotheses, and as irresistibly compelling assent, *on condition* that those hypotheses are true.

When, therefore, it is affirmed that the conclusions of geometry are necessary truths, the necessity consists in reality only in this, that they necessarily follow from the suppositions from which they are deduced. Those suppositions are so far from being necessary, that they are not even true; they purposely depart, more or less widely, from the truth. The only sense in which necessity can be ascribed to the conclusions of any scientific investigation, is that of necessarily following from some assumption, which, by the conditions of the inquiry, is not to be questioned. In this relation, of course, the derivative truths of every deductive science must stand to the inductions, or assumptions, on which the science is founded, and which, whether true or untrue, certain or doubtful in themselves, are always supposed certain for the purposes of the particular science. And therefore the conclusions of all deductive sciences were said by the ancients to be necessary propositions. We have observed already that to be predicated necessarily was characteristic of the predicable Proprium, and that a proprium was any property of a thing which could be deduced from its essence, that is, from the properties included in its definition.

§ 2. The important doctrine of Dugald Stewart, which I have endeavoured to enforce, has been contested by Dr. Whewell, both in the dissertation appended to his excellent *Mechanical Euclid*, and in his more recent elaborate work on the *Philosophy of the Inductive Sciences*; in which last he also replies to an article in the *Edinburgh Review*, (ascribed to a writer of great scientific eminence,) in which Stewart's opinion was defended against his former strictures. The supposed refutation of Stewart consists in proving against him (as has also been done in this work) that the premisses of geometry are not definitions, but assumptions of the real existence of things corresponding to those definitions. This, however, is doing little for Dr. Whewell's purpose; for it is these very assumptions which are asserted to be hypotheses, and which he, if he denies that geometry is founded on hypotheses, must show to be absolute truths. All he does, however, is to observe, that they at any rate are not *arbitrary* hypotheses; that we should not be at liberty to substitute other hypotheses for them; that not only "a definition, to be admissible, must necessarily refer to and agree with some conception which we can distinctly frame in our thoughts," but that the straight lines, for instance, which we define, must be "those by which angles are contained, those by which triangles are bounded, those of which parallelism may be predicated, and the like."(37) And this is true; but this has never been contradicted. Those who say that the premisses of geometry are hypotheses, are not bound to maintain them to be hypotheses which have no relation whatever to fact. Since an hypothesis framed for the purpose of scientific inquiry must relate to something which has real existence, (for there can be no science respecting non-entities,) it follows that any hypothesis we make respecting an object, to facilitate our study of it, must not involve anything which is distinctly false, and repugnant to its real nature: we must not ascribe to the thing any property which it has not; our liberty extends only to suppressing some of those which it has, under the indispensable obligation of restoring them whenever, and in as far as, their presence or absence would make any material difference in the truth of our conclusions. Of this nature, accordingly, are the first principles involved in the definitions of geometry. In their positive part they are observed facts; it is only in their negative part that they are hypothetical. That the hypotheses should be of this particular character, is however no further necessary, than inasmuch as no others could enable us to deduce conclusions which, with due corrections, would be true of real objects: and in fact, when our aim is only to illustrate truths, and not to investigate them, we are not under any such restriction. We might suppose an imaginary animal, and work out by deduction, from the known laws of physiology, its natural history; or an imaginary commonwealth, and from the elements composing it, might argue what would be its fate. And the conclusions which we might thus draw from purely arbitrary hypotheses, might form a highly useful intellectual exercise: but as they could only teach us what *would* be the properties of objects which do not really exist, they would not constitute any addition to our knowledge of nature: while on the contrary, if the hypothesis merely divests a real object of some portion of its properties, without clothing it in false ones, the conclusions will always express, under known liability to correction, actual truth.

§ 3. But although Dr. Whewell has not shaken Stewart's doctrine as to the hypothetical character of that portion of the first principles of geometry which are involved in the so-called definitions, he has, I conceive, greatly the advantage of Stewart on another important point in the theory of geometrical reasoning; the necessity of admitting, among those first principles, axioms as well as definitions. Some of the axioms of Euclid might, no doubt, be exhibited in the form of definitions, or might be deduced, by reasoning, from propositions similar to what are so called. Thus, if instead of the axiom, Magnitudes which can be made to coincide are equal, we introduce a definition, "Equal magnitudes are those which may be so applied to one another as to coincide;" the three axioms which follow, (Magnitudes which are equal to the same are equal to one another--If equals are added to equals the sums are equal--If equals are taken from equals the remainders are equal,) may be proved by an imaginary superposition, resembling that by which the fourth proposition of the first book of Euclid is demonstrated. But although these and several others may be struck out of the list of first principles, because, though not requiring demonstration, they are susceptible of it; there will be found in the list of axioms two or three fundamental truths, not capable of being demonstrated: among which must be reckoned the proposition that two straight lines cannot inclose a space, (or its equivalent, Straight lines which coincide in two points coincide altogether,) and some property of parallel lines, other than that which constitutes their definition: the most suitable, perhaps, being that selected by Professor Playfair: "Two straight lines which intersect each other cannot both of them be parallel to a third straight line."(38)

The axioms, as well those which are indemonstrable as those which admit of being demonstrated, differ from that other class of fundamental principles which are involved in the definitions, in this, that they are true without any mixture of hypothesis. That things which are equal to the same thing are equal to one another, is as true of the lines and figures in nature, as it would be of the imaginary ones assumed in the definitions. In this respect, however, mathematics are only on a par with most other sciences. In almost all sciences there are some general propositions which are exactly true, while the greater part are only more or less distant approximations to the truth. Thus in mechanics, the first law of motion (the continuance of a movement once impressed, until stopped or slackened by some resisting force) is true without qualification or error. The rotation of the earth in twenty-four hours, of the same length as in our time, has gone on since the first accurate observations, without the increase or diminution of one second in all that period. These are inductions which require no fiction to make them be received as accurately true: but along with them there are others, as for instance the propositions respecting the figure of the earth, which are but approximations to the truth; and in order to use them for the further advancement of our knowledge, we must feign that they are exactly true, though they really want something of being so.

§ 4. It remains to inquire, what is the ground of our belief in axioms--what is the evidence on which they rest? I answer, they are experimental truths; generalizations from observation. The proposition, Two straight lines cannot inclose a space--or in other words, Two straight lines which have once met, do not meet again, but continue to diverge--is an induction from the evidence of our senses.

This opinion runs counter to a scientific prejudice of long standing and great strength, and there is probably no one proposition enunciated in this work for which a more unfavourable reception is to be expected. It is, however, no new opinion; and even if it were so, would be entitled to be judged, not by its novelty, but by the strength of the arguments by which it can be supported. I consider it very fortunate that so eminent a champion of the contrary opinion as Dr. Whewell, has recently found occasion for a most elaborate treatment of the whole theory of axioms, in attempting to construct the philosophy of the mathematical and physical sciences on the basis of the doctrine against which I now contend. Whoever is anxious that a discussion should go to the bottom of the subject, must rejoice to see the opposite side of the question worthily represented. If what is said by Dr. Whewell, in support of an opinion which he has made the foundation of a systematic work, can be shown not to be conclusive, enough will have been done without going further to seek stronger arguments and a more powerful adversary.

It is not necessary to show that the truths which we call axioms are originally *suggested* by observation, and that we should never have known that two straight lines cannot inclose a space if we had never seen a straight line: thus much being admitted by Dr. Whewell, and by all, in recent times, who have taken his view of the subject. But they contend, that it is not experience which *proves* the axiom; but that its truth is perceived *à priori*, by the constitution of the mind itself, from the first moment when the meaning of the proposition is apprehended; and without any necessity for verifying it by repeated trials, as is requisite in the case of truths really ascertained by observation.

They cannot, however, but allow that the truth of the axiom, Two straight lines cannot inclose a space, even if evident independently of experience, is also evident from experience. Whether the axiom *needs* confirmation or not, it *receives* confirmation in almost every instant of our lives; since we cannot look at any two straight lines which intersect one another, without seeing that from that point they continue to diverge more and more. Experimental proof crowds in upon us in such endless profusion, and without one instance in which there can be even a suspicion of an exception to the rule, that we should soon have a stronger ground for believing the axiom, even as an experimental truth, than we have for almost any of the general truths which we confessedly learn from the evidence of our senses. Independently of *à priori* evidence, we should certainly believe it with an intensity of conviction far greater than we accord to any ordinary physical truth: and this too at a time of life much earlier than that from which we date almost any part of our acquired knowledge, and much too early to admit of our retaining any recollection of the history of our intellectual operations at that period. Where then is the necessity for assuming that our recognition of these truths has a different origin from the rest of our knowledge, when its existence is perfectly accounted for by supposing its origin to be the same? when the causes which produce belief in all other instances, exist in this instance, and in a degree of strength as much superior to what exists in other cases, as the intensity of the belief itself is superior? The burden of proof lies on the advocates of the contrary opinion: it is for them to point out some fact, inconsistent with the supposition that this part of our knowledge of nature is derived from the same sources as every other part.

This, for instance, they would be able to do, if they could prove chronologically that we had the conviction (at least practically) so early in infancy as to be anterior to those impressions on the senses, upon which, on the other theory, the conviction is founded. This, however, cannot be proved: the point being too far back to be within the reach of memory, and too obscure for external observation. The advocates of the *à priori* theory are obliged to have recourse to other arguments. These are reducible to two, which I shall endeavour to state as clearly and as forcibly as possible.

§ 5. In the first place it is said, that if our assent to the proposition that two straight lines cannot inclose a space, were derived from the senses, we could only be convinced of its truth by actual trial, that is, by seeing or feeling the straight lines; whereas in fact it is seen to be true by merely thinking of them. That a stone thrown into water goes to the bottom, may be perceived by our senses, but mere thinking of a stone thrown into the water would never have led us to that conclusion: not so, however, with the axioms relating to straight lines: if I could be made to conceive what a straight line is, without having seen one, I should at once recognise that two such lines cannot inclose a space. Intuition is "imaginary looking;"(39) but experience must be real looking: if we see a property of straight lines to be true by merely fancying ourselves to be looking at them, the ground of our belief cannot be the senses, or experience; it must be something mental.

To this argument it might be added in the case of this particular axiom, (for the assertion would not be true of all axioms,) that the evidence of it from actual ocular inspection, is not only unnecessary, but unattainable. What says the axiom? That two straight lines *cannot* inclose a space; that after having once intersected, if they are prolonged to infinity they do not meet, but continue to diverge from one another. How can this, in any single case, be proved by actual observation? We may follow the lines to any distance we please; but we cannot follow them to infinity: for aught our senses can testify, they may, immediately beyond the farthest point to which we have traced them, begin to approach, and at last meet. Unless, therefore, we had some other proof of the impossibility than observation affords us, we should have no ground for believing the axiom at all.

To these arguments, which I trust I cannot be accused of understating, a satisfactory answer will, I conceive, be found, if we advert to one of the characteristic properties of geometrical forms--their capacity of being painted in the imagination with a distinctness equal to reality: in other words, the exact resemblance of our ideas of form to the sensations which suggest them. This, in the first place, enables us to make (at least with a little practice) mental pictures of all possible combinations of lines and angles, which resemble the realities quite as well as any which we could make on paper; and in the next place, makes those pictures just as fit subjects of geometrical experimentation as the realities themselves; inasmuch as pictures, if sufficiently accurate, exhibit of course all the properties which would be manifested by the realities at one given instant, and on simple inspection: and in geometry we are concerned only with such properties, and not with that which pictures could not exhibit, the mutual action of bodies one upon another. The foundations of geometry would therefore be laid in direct experience, even if the experiments (which in this case consist merely in attentive contemplation) were practised solely upon what we call our ideas, that is, upon the diagrams in our minds, and not upon outward objects. For in all systems of experimentation we take some objects to serve as representatives of all which resemble them; and in the present case the conditions which qualify a real object to be the representative of its class, are completely fulfilled by an object existing only in our fancy. Without denying, therefore, the possibility of satisfying ourselves that two straight lines cannot inclose a space, by merely thinking of straight lines without actually looking at them; I contend, that we do not believe this truth on the ground of the imaginary intuition simply, but because we know that the imaginary lines exactly resemble real ones, and that we may conclude from them to real ones with quite as much certainty as we could conclude from one real line to another. The conclusion, therefore, is still an induction from observation. And we should not be authorized to substitute observation of the image in our mind, for observation of the reality, if we had not learnt by long-continued experience that the properties of the reality are faithfully represented in the image; just as we should be scientifically warranted in describing an animal which we had never seen, from a picture made of it with a daguerreotype; but not until we had learnt by ample experience, that observation of such a picture is precisely equivalent to observation of the original.

These considerations also remove the objection arising from the impossibility of ocularly following the lines in their prolongation to infinity, for though, in order actually to see that two given lines never meet, it would be necessary to follow them to infinity; yet without doing so we may know that if they ever do meet, or if, after diverging from one another, they begin again to approach, this must take place not at an infinite, but at a finite distance. Supposing, therefore, such to be the case, we can transport ourselves thither in imagination, and can frame a mental image of the appearance which one or both of the lines must present at that point, which we may rely on as being precisely similar to the reality. Now, whether we fix our contemplation upon this imaginary picture, or call to mind the generalizations we have had occasion to make from former ocular observation, we learn by the evidence of experience, that a line which, after diverging from another straight line, begins to approach to it, produces the impression on our senses which we describe by the expression, "a bent line," not by the expression, "a straight line."(40)

§ 6. The first of the two arguments in support of the theory that axioms are *à priori* truths, having, I think, been sufficiently answered; I proceed to the second, which is usually the most relied on. Axioms (it is asserted) are conceived by us not only as true, but as universally and necessarily true. Now, experience cannot possibly give to any proposition this character. I may have seen snow a hundred times, and may have seen that it was white, but this cannot give me entire assurance even that all snow is white; much less that snow *must* be white. "However many instances we may have observed of the truth of a proposition, there is nothing to assure us that the next case shall not be an exception to the rule. If it be strictly true that every ruminant animal yet known has cloven hoofs, we still cannot be sure that some creature will not hereafter be discovered which has the first of these attributes, without having the other.... Experience must always consist of a limited number of observations; and, however numerous these may be, they can show nothing with regard to the infinite number of cases in which the experiment has not been made." Besides, axioms are not only universal, they are also necessary. Now "experience cannot offer the smallest ground for the necessity of a proposition. She can observe and record what has happened; but she cannot find, in any case, or in any accumulation of cases, any reason for what *must* happen. She may see objects side by side; but she cannot see a reason why they must ever be side by side. She finds certain events to occur in succession; but the succession supplies, in its occurrence, no reason for its recurrence. She contemplates external objects; but she cannot detect any internal bond, which indissolubly connects the future with the past, the possible with the real. To learn a proposition by experience, and to see it to be necessarily true, are two altogether different processes of thought."(41) And Dr. Whewell adds, "If any one does not clearly comprehend this distinction of necessary and contingent truths, he will not be able to go along with us in our researches into the foundations of human knowledge; nor, indeed, to pursue with success any speculation on the subject."(42)

In the following passage, we are told what the distinction is, the non-recognition of which incurs this denunciation. "Necessary truths are those in which we not only learn that the proposition *is* true, but see that it *must be* true; in which the negation of the truth is not only false, but impossible; in which we cannot, even by an effort of imagination, or in a supposition, conceive the reverse of that which is asserted. That there are such truths cannot be doubted. We may take, for example, all relations of number. Three and Two, added together, make Five. We cannot conceive it to be otherwise. We cannot, by any freak of thought, imagine Three and Two to make Seven."(43)

Although Dr. Whewell has naturally and properly employed a variety of phrases to bring his meaning more forcibly home, he will, I presume, allow that they are all equivalent; and that what he means by a necessary truth, would be sufficiently defined, a proposition the negation of which is not only false but inconceivable. I am unable to find in any of his expressions, turn them what way you will, a meaning beyond this, and I do not believe he would contend that they mean anything more.

This, therefore, is the principle asserted: that propositions, the negation of which is inconceivable, or in other words, which we cannot figure to ourselves as being false, must rest on evidence of a higher and more cogent description than any which experience can afford. And we have next to consider whether there is any ground for this assertion.

Now I cannot but wonder that so much stress should be laid on the circumstance of inconceivableness, when there is such ample experience to show, that our capacity or incapacity of conceiving a thing has very little to do with the possibility of the thing in itself; but is in truth very much an affair of accident, and depends on the past history and habits of our own minds. There is no more generally acknowledged fact in human nature, than the extreme difficulty at first felt in conceiving anything as possible, which is in contradiction to long established and familiar experience; or even to old familiar habits of thought. And this difficulty is a necessary result of the fundamental laws of the human mind. When we have often seen and thought of two things together, and have never in any one instance either seen or thought of them separately, there is by the primary law of association an increasing difficulty, which may in the end become insuperable, of conceiving the two things apart. This is most of all conspicuous in uneducated persons, who are in general utterly unable to separate any two ideas which have once become firmly associated in their minds; and if persons of cultivated intellect have any advantage on the point, it is only because, having seen and heard and read more, and being more accustomed to exercise their imagination, they have experienced their sensations and thoughts in more varied combinations, and have been prevented from forming many of these inseparable associations. But this advantage has necessarily its limits. The most practised intellect is not exempt from the universal laws of our conceptive faculty. If daily habit presents to any one for a long period two facts in combination, and if he is not led during that period either by accident or by his voluntary mental operations to think of them apart, he will probably in time become incapable of doing so even by the strongest effort; and the supposition that the two facts can be separated in nature, will at last present itself to his mind with all the characters of an inconceivable phenomenon.(44) There are remarkable instances of this in the history of science: instances in which the most instructed men rejected as impossible, because inconceivable, things which their posterity, by earlier practice and longer perseverance in the attempt, found it quite easy to conceive, and which everybody now knows to be true. There was a time when men of the most cultivated intellects, and the most emancipated from the dominion of early prejudice, could not credit the existence of antipodes; were unable to conceive, in opposition to old association, the force of gravity acting upwards instead of downwards. The Cartesians long rejected the Newtonian doctrine of the gravitation of all bodies towards one another, on the faith of a general proposition, the reverse of which seemed to them to be inconceivable--the proposition that a body cannot act where it is not. All the cumbrous machinery of imaginary vortices, assumed without the smallest particle of evidence, appeared to these philosophers a more rational mode of explaining the heavenly motions, than one which involved what seemed to them so great an absurdity.(45) And they no doubt found it as impossible to conceive that a body should act upon the earth, at the distance of the sun or moon, as we find it to conceive an end to space or time, or two straight lines inclosing a space. Newton himself had not been able to realize the conception, or we should not have had his hypothesis of a subtle ether, the occult cause of gravitation; and his writings prove, that although he deemed the particular nature of the intermediate agency a matter of conjecture, the necessity of *some* such agency appeared to him indubitable. It would seem that even now the majority of scientific men have not completely got over this very difficulty; for though they have at last learnt to conceive the sun *attracting* the earth without any intervening fluid, they cannot yet conceive the sun *illuminating* the earth without some such medium.

If, then, it be so natural to the human mind, even in a high state of culture, to be incapable of conceiving, and on that ground to believe impossible, what is afterwards not only found to be conceivable but proved to be true; what wonder if in cases where the association is still older, more confirmed, and more familiar, and in which nothing ever occurs to shake our conviction, or even suggest to us any conception at variance with the association, the acquired incapacity should continue, and be mistaken for a natural incapacity? It is true, our experience of the varieties in nature enables us, within certain limits, to conceive other varieties analogous to them. We can conceive the sun or moon falling; for although we never saw them fall, nor ever perhaps imagined them falling, we have seen so many other things fall, that we have innumerable familiar analogies to assist the conception; which, after all, we should probably have some difficulty in framing, were we not well accustomed to see the sun and moon move, (or appear to move,) so that we are only called upon to conceive a slight change in the direction of motion, a circumstance familiar to our experience. But when experience affords no model on which to shape the new conception, how is it possible for us to form it? How, for example, can we imagine an end to space or time? We never saw any object without something beyond it, nor experienced any feeling without something following it. When, therefore, we attempt to conceive the last point of space, we have the idea irresistibly raised of other points beyond it. When we try to imagine the last instant of time, we cannot help conceiving another instant after it. Nor is there any necessity to assume, as is done by a modern school of metaphysicians, a peculiar fundamental law of the mind to account for the feeling of infinity inherent in our conceptions of space and time; that apparent infinity is sufficiently accounted for by simpler and universally acknowledged laws.

Now, in the case of a geometrical axiom, such, for example, as that two straight lines cannot inclose a space,--a truth which is testified to us by our very earliest impressions of the external world,--how is it possible (whether those external impressions be or be not the ground of our belief) that the reverse of the proposition *could* be otherwise than inconceivable to us? What analogy have we, what similar order of facts in any other branch of our experience, to facilitate to us the conception of two straight lines inclosing a space? Nor is even this all. I have already called attention to the peculiar property of our impressions of form, that the ideas or mental images exactly resemble their prototypes, and adequately represent them for the purposes of scientific observation. From this, and from the intuitive character of the observation, which in this case reduces itself to simple inspection, we cannot so much as call up in our imagination two straight lines, in order to attempt to conceive them inclosing a space, without by that very act repeating the scientific experiment which establishes the contrary. Will it really be contended that the inconceivableness of the thing, in such circumstances, proves anything against the experimental origin of the conviction? Is it not clear that in whichever mode our belief in the proposition may have originated, the impossibility of our conceiving the negative of it must, on either hypothesis, be the same? As, then, Dr. Whewell exhorts those who have any difficulty in recognising the distinction held by him between necessary and contingent truths, to study geometry,--a condition which I can assure him I have conscientiously fulfilled,--I, in return, with equal confidence, exhort those who agree with him, to study the elementary laws of association; being convinced that nothing more is requisite than a moderate familiarity with those laws, to dispel the illusion which ascribes a peculiar necessity to our earliest inductions from experience, and measures the possibility of things in themselves, by the human capacity of conceiving them.

I hope to be pardoned for adding, that Dr. Whewell himself has both confirmed by his testimony the effect of habitual association in giving to an experimental truth the appearance of a necessary one, and afforded a striking instance of that remarkable law in his own person. In his *Philosophy of the Inductive Sciences* he continually asserts, that propositions which not only are not self-evident, but which we know to have been discovered gradually, and by great efforts of genius and patience, have, when once established, appeared so self-evident that, but for historical proof, it would have been impossible to conceive that they had not been recognised from the first by all persons in a sound state of their faculties. "We now despise those who, in the Copernican controversy, could not conceive the apparent motion of the sun on the heliocentric hypothesis; or those who, in opposition to Galileo, thought that a uniform force might be that which generated a velocity proportional to the space; or those who held there was something absurd in Newton's doctrine of the different refrangibility of differently coloured rays; or those who imagined that when elements combine, their sensible qualities must be manifest in the compound; or those who were reluctant to give up the distinction of vegetables into herbs, shrubs, and trees. We cannot help thinking that men must have been singularly dull of comprehension to find a difficulty in admitting what is to us so plain and simple. We have a latent persuasion that we in their place should have been wiser and more clearsighted; that we should have taken the right side, and given our assent at once to the truth. Yet in reality such a persuasion is a mere delusion. The persons who, in such instances as the above, were on the losing side, were very far in most cases from being persons more prejudiced, or stupid, or narrow-minded, than the greater part of mankind now are; and the cause for which they fought was far from being a manifestly bad one, till it had been so decided by the result of the war.... So complete has been the victory of truth in most of these instances, that at present we can hardly imagine the struggle to have been necessary. *The very essence of these triumphs is, that they lead us to regard the views we reject as not only false but inconceivable.*"(46)

This last proposition is precisely what I contend for; and I ask no more, in order to overthrow the whole theory of its author on the nature of the evidence of axioms. For what is that theory? That the truth of axioms cannot have been learnt from experience, because their falsity is inconceivable. But Dr. Whewell himself says, that we are continually led by the natural progress of thought, to regard as inconceivable what our forefathers not only conceived but believed, nay even (he might have added) were unable to conceive the contrary of. He cannot intend to justify this mode of thought: he cannot mean to say, that we can be *right* in regarding as inconceivable what others have conceived, and as self-evident what to others did not appear evident at all. After so complete an admission that inconceivableness is an accidental thing, not inherent in the phenomenon itself, but dependent on the mental history of the person who tries to conceive it, how can he ever call upon us to reject a proposition as impossible on no other ground than its inconceivableness? Yet he not only does so, but has unintentionally afforded some of the most remarkable examples which can be cited of the very illusion which he has himself so clearly pointed out. I select as specimens, his remarks on the evidence of the three laws of motion, and of the atomic theory.

With respect to the laws of motion, Dr. Whewell says: "No one can doubt that, in historical fact, these laws were collected from experience. That such is the case, is no matter of conjecture. We know the time, the persons, the circumstances, belonging to each step of each discovery."(47) After this testimony, to adduce evidence of the fact would be superfluous. And not only were these laws by no means intuitively evident, but some of them were originally paradoxes. The first law was especially so. That a body, once in motion, would continue for ever to move in the same direction with undiminished velocity unless acted upon by some new force, was a proposition which mankind found for a long time the greatest difficulty in crediting. It stood opposed to apparent experience of the most familiar kind, which taught that it was the nature of motion to abate gradually, and at last terminate of itself. Yet when once the contrary doctrine was firmly established, mathematicians, as Dr. Whewell observes, speedily began to believe that laws, thus contradictory to first appearances, and which, even after full proof had been obtained, it had required generations to render familiar to the minds of the scientific world, were under "a demonstrable necessity, compelling them to be such as they are and no other;" and he himself, though not venturing "absolutely to pronounce" that *all* these laws "can be rigorously traced to an absolute necessity in the nature of things,"(48) does actually think in that manner of the law just mentioned; of which he says: "Though the discovery of the first law of motion was made, historically speaking, by means of experiment, we have now attained a point of view in which we see that it might have been certainly known to be true, independently of experience."(49) Can there be a more striking exemplification than is here afforded, of the effect of association which we have described? Philosophers, for generations, have the most extraordinary difficulty in putting certain ideas together; they at last succeed in doing so; and after a sufficient repetition of the process, they first fancy a natural bond between the ideas, then experience a growing difficulty, which at last, by the continuation of the same progress, becomes an impossibility, of severing them from one another. If such be the progress of an experimental conviction of which the date is of yesterday, and which is in opposition to first appearances, how must it fare with those which are conformable to appearances familiar from the first dawn of intelligence, and of the conclusiveness of which, from the earliest records of human thought, no sceptic has suggested even a momentary doubt?

The other instance which I shall quote is a truly astonishing one, and may be called the *reductio ad absurdum* of the theory of inconceivableness. Speaking of the laws of chemical composition, Dr. Whewell says:(50) "That they could never have been clearly understood, and therefore never firmly established, without laborious and exact experiments, is certain; but yet we may venture to say, that being once known, they possess an evidence beyond that of mere experiment. *For how, in fact, can we conceive combinations, otherwise than as definite in kind and quality?* If we were to suppose each element ready to combine with any other indifferently, and indifferently in any quantity, we should have a world in which all would be confusion and indefiniteness. There would be no fixed kinds of bodies; salts, and stones, and ores, would approach to and graduate into each other by insensible degrees. Instead of this, we know that the world consists of bodies distinguishable from each other by definite differences, capable of being classified and named, and of having general propositions asserted concerning them. And as *we cannot conceive a world in which this should not be the case*, it would appear that we cannot conceive a state of things in which the laws of the combination of elements should not be of that definite and measured kind which we have above asserted."(51)

That a philosopher of Dr. Whewell's eminence should gravely assert that we cannot conceive a world in which the simple elements would combine in other than definite proportions; that by dint of meditating on a scientific truth, the original discoverer of which was still living, he should have rendered the association in his own mind between the idea of combination and that of constant proportions so familiar and intimate as to be unable to conceive the one fact without the other; is so signal an instance of the mental law for which I am contending, that one word more in illustration must be superfluous.(52)