Chapter IV.

Of Trains Of Reasoning, And Deductive Sciences.

§ 1. In our analysis of the syllogism, it appeared that the minor premise always affirms a resemblance between a new case and some cases previously known; while the major premise asserts something which, having been found true of those known cases, we consider ourselves warranted in holding true of any other case resembling the former in certain given particulars.

If all ratiocinations resembled, as to the minor premise, the examples which were exclusively employed in the preceding chapter; if the resemblance, which that premise asserts, were obvious to the senses, as in the proposition "Socrates is a man," or were at once ascertainable by direct observation; there would be no necessity for trains of reasoning, and Deductive or Ratiocinative Sciences would not exist. Trains of reasoning exist only for the sake of extending an induction founded, as all inductions must be, on observed cases, to other cases in which we not only can not directly observe the fact which is to be proved, but can not directly observe even the mark which is to prove it.

§ 2. Suppose the syllogism to be, All cows ruminate, the animal which is before me is a cow, therefore it ruminates. The minor, if true at all, is obviously so: the only premise the establishment of which requires any anterior process of inquiry, is the major; and provided the induction of which that premise is the expression was correctly performed, the conclusion respecting the animal now present will be instantly drawn; because, as soon as she is compared with the formula, she will be identified as being included in it. But suppose the syllogism to be the following: All arsenic is poisonous, the substance which is before me is arsenic, therefore it is poisonous. The truth of the minor may not here be obvious at first sight; it may not be intuitively evident, but may itself be known only by inference. It may be the conclusion of another argument, which, thrown into the syllogistic form, would stand thus: Whatever when lighted produces a dark spot on a piece of white porcelain held in the flame, which spot is soluble in hypochloride of calcium, is arsenic; the substance before me conforms to this condition; therefore it is arsenic. To establish, therefore, the ultimate conclusion, The substance before me is poisonous, requires a process, which, in order to be syllogistically expressed, stands in need of two syllogisms; and we have a Train of Reasoning.

When, however, we thus add syllogism to syllogism, we are really adding induction to induction. Two separate inductions must have taken place to render this chain of inference possible; inductions founded, probably, on different sets of individual instances, but which converge in their results, so that the instance which is the subject of inquiry comes within the range of them both. The record of these inductions is contained in the majors of the two syllogisms. First, we, or others for us, have examined various objects which yielded under the given circumstances a dark spot with the given property, and found that they possessed the properties connoted by the word arsenic; they were metallic, volatile, their vapor had a smell of garlic, and so forth. Next, we, or others for us, have examined various specimens which possessed this metallic and volatile character, whose vapor had this smell, etc., and have invariably found that they were poisonous. The first observation we judge that we may extend to all substances whatever which yield that particular kind of dark spot; the second, to all metallic and volatile substances resembling those we examined; and consequently, not to those only which are seen to be such, but to those which are concluded to be such by the prior induction. The substance before us is only seen to come within one of these inductions; but by means of this one, it is brought within the other. We are still, as before, concluding from particulars to particulars; but we are now concluding from particulars observed, to other particulars which are not, as in the simple case, *seen* to resemble them in material points, but *inferred* to do so, because resembling them in something else, which we have been led by quite a different set of instances to consider as a mark of the former resemblance.

This first example of a train of reasoning is still extremely simple, the series consisting of only two syllogisms. The following is somewhat more complicated: No government, which earnestly seeks the good of its subjects, is likely to be overthrown; some particular government earnestly seeks the good of its subjects, therefore it is not likely to be overthrown. The major premise in this argument we shall suppose not to be derived from considerations *a priori*, but to be a generalization from history, which, whether correct or erroneous, must have been founded on observation of governments concerning whose desire of the good of their subjects there was no doubt. It has been found, or thought to be found, that these were not easily overthrown, and it has been deemed that those instances warranted an extension of the same predicate to any and every government which resembles them in the attribute of desiring earnestly the good of its subjects. But *does* the government in question thus resemble them? This may be debated *pro* and *con* by many arguments, and must, in any case, be proved by another induction; for we can not directly observe the sentiments and desires of the persons who carry on the government. To prove the minor, therefore, we require an argument in this form: Every government which acts in a certain manner, desires the good of its subjects; the supposed government acts in that particular manner, therefore it desires the good of its subjects. But is it true that the government acts in the manner supposed? This minor also may require proof; still another induction, as thus: What is asserted by intelligent and disinterested witnesses, may be believed to be true; that the government acts in this manner, is asserted by such witnesses, therefore it may be believed to be true. The argument hence consists of three steps. Having the evidence of our senses that the case of the government under consideration resembles a number of former cases, in the circumstance of having something asserted respecting it by intelligent and disinterested witnesses, we infer, first, that, as in those former instances, so in this instance, the assertion is true. Secondly, what was asserted of the government being that it acts in a particular manner, and other governments or persons having been observed to act in the same manner, the government in question is brought into known resemblance with those other governments or persons; and since they were known to desire the good of the people, it is thereupon, by a second induction, inferred that the particular government spoken of, desires the good of the people. This brings that government into known resemblance with the other governments which were thought likely to escape revolution, and thence, by a third induction, it is concluded that this particular government is also likely to escape. This is still reasoning from particulars to particulars, but we now reason to the new instance from three distinct sets of former instances: to one only of those sets of instances do we directly perceive the new one to be similar; but from that similarity we inductively infer that it has the attribute by which it is assimilated to the next set, and brought within the corresponding induction; after which by a repetition of the same operation we infer it to be similar to the third set, and hence a third induction conducts us to the ultimate conclusion.

§ 3. Notwithstanding the superior complication of these examples, compared with those by which in the preceding chapter we illustrated the general theory of reasoning, every doctrine which we then laid down holds equally true in these more intricate cases. The successive general propositions are not steps in the reasoning, are not intermediate links in the chain of inference, between the particulars observed and those to which we apply the observation. If we had sufficiently capacious memories, and a sufficient power of maintaining order among a huge mass of details, the reasoning could go on without any general propositions; they are mere formulæ for inferring particulars from particulars. The principle of general reasoning is (as before explained), that if, from observation of certain known particulars, what was seen to be true of them can be inferred to be true of any others, it may be inferred of all others which are of a certain description. And in order that we may never fail to draw this conclusion in a new case when it can be drawn correctly, and may avoid drawing it when it can not, we determine once for all what are the distinguishing marks by which such cases may be recognized. The subsequent process is merely that of identifying an object, and ascertaining it to have those marks; whether we identify it by the very marks themselves, or by others which we have ascertained (through another and a similar process) to be marks of those marks. The real inference is always from particulars to particulars, from the observed instances to an unobserved one: but in drawing this inference, we conform to a formula which we have adopted for our guidance in such operations, and which is a record of the criteria by which we thought we had ascertained that we might distinguish when the inference could, and when it could not, be drawn. The real premises are the individual observations, even though they may have been forgotten, or, being the observations of others and not of ourselves, may, to us, never have been known: but we have before us proof that we or others once thought them sufficient for an induction, and we have marks to show whether any new case is one of those to which, if then known, the induction would have been deemed to extend. These marks we either recognize at once, or by the aid of other marks, which by another previous induction we collected to be marks of the first. Even these marks of marks may only be recognized through a third set of marks; and we may have a train of reasoning, of any length, to bring a new case within the scope of an induction grounded on particulars its similarity to which is only ascertained in this indirect manner.

Thus, in the preceding example, the ultimate inductive inference was, that a certain government was not likely to be overthrown; this inference was drawn according to a formula in which desire of the public good was set down as a mark of not being likely to be overthrown; a mark of this mark was, acting in a particular manner; and a mark of acting in that manner was, being asserted to do so by intelligent and disinterested witnesses: this mark, the government under discussion was recognized by the senses as possessing. Hence that government fell within the last induction, and by it was brought within all the others. The perceived resemblance of the case to one set of observed particular cases, brought it into known resemblance with another set, and that with a third.

In the more complex branches of knowledge, the deductions seldom consist, as in the examples hitherto exhibited, of a single chain, *a* a mark of *b, b* of *c, c* of *d*, therefore *a* a mark of *d*. They consist (to carry on the same metaphor) of several chains united at the extremity, as thus: *a* a mark of *d, b* of *e, c* of *f, d e f* of *n*, therefore *a b c* a mark of *n*. Suppose, for example, the following combination of circumstances: 1st, rays of light impinging on a reflecting surface; 2d, that surface parabolic; 3d, those rays parallel to each other and to the axis of the surface. It is to be proved that the concourse of these three circumstances is a mark that the reflected rays will pass through the focus of the parabolic surface. Now, each of the three circumstances is singly a mark of something material to the case. Rays of light impinging on a reflecting surface are a mark that those rays will be reflected at an angle equal to the angle of incidence. The parabolic form of the surface, is a mark that, from any point of it, a line drawn to the focus and a line parallel to the axis will make equal angles with the surface. And finally, the parallelism of the rays to the axis is a mark that their angle of incidence coincides with one of these equal angles. The three marks taken together are therefore a mark of all these three things united. But the three united are evidently a mark that the angle of reflection must coincide with the other of the two equal angles, that formed by a line drawn to the focus; and this again, by the fundamental axiom concerning straight lines, is a mark that the reflected rays pass through the focus. Most chains of physical deduction are of this more complicated type; and even in mathematics such are abundant, as in all propositions where the hypothesis includes numerous conditions: "*If* a circle be taken, and *if* within that circle a point be taken, not the centre, and *if* straight lines be drawn from that point to the circumference, then," etc.

§ 4. The considerations now stated remove a serious difficulty from the view we have taken of reasoning; which view might otherwise have seemed not easily reconcilable with the fact that there are Deductive or Ratiocinative Sciences. It might seem to follow, if all reasoning be induction, that the difficulties of philosophical investigation must lie in the inductions exclusively, and that when these were easy, and susceptible of no doubt or hesitation, there could be no science, or, at least, no difficulties in science. The existence, for example, of an extensive Science of Mathematics, requiring the highest scientific genius in those who contributed to its creation, and calling for a most continued and vigorous exertion of intellect in order to appropriate it when created, may seem hard to be accounted for on the foregoing theory. But the considerations more recently adduced remove the mystery, by showing, that even when the inductions themselves are obvious, there may be much difficulty in finding whether the particular case which is the subject of inquiry comes within them; and ample room for scientific ingenuity in so combining various inductions, as, by means of one within which the case evidently falls, to bring it within others in which it can not be directly seen to be included.

When the more obvious of the inductions which can be made in any science from direct observations, have been made, and general formulas have been framed, determining the limits within which these inductions are applicable; as often as a new case can be at once seen to come within one of the formulas, the induction is applied to the new case, and the business is ended. But new cases are continually arising, which do not obviously come within any formula whereby the question we want solved in respect of them could be answered. Let us take an instance from geometry: and as it is taken only for illustration, let the reader concede to us for the present, what we shall endeavor to prove in the next chapter, that the first principles of geometry are results of induction. Our example shall be the fifth proposition of the first book of Euclid. The inquiry is, Are the angles at the base of an isosceles triangle equal or unequal? The first thing to be considered is, what inductions we have, from which we can infer equality or inequality. For inferring equality we have the following formulæ: Things which being applied to each other coincide, are equals. Things which are equal to the same thing are equals. A whole and the sum of its parts are equals. The sums of equal things are equals. The differences of equal things are equals. There are no other original formulæ to prove equality. For inferring inequality we have the following: A whole and its parts are unequals. The sums of equal things and unequal things are unequals. The differences of equal things and unequal things are unequals. In all, eight formulæ. The angles at the base of an isosceles triangle do not obviously come within any of these. The formulæ specify certain marks of equality and of inequality, but the angles can not be perceived intuitively to have any of those marks. On examination it appears that they have; and we ultimately succeed in bringing them within the formula, "The differences of equal things are equal." Whence comes the difficulty of recognizing these angles as the differences of equal things? Because each of them is the difference not of one pair only, but of innumerable pairs of angles; and out of these we had to imagine and select two, which could either be intuitively perceived to be equals, or possessed some of the marks of equality set down in the various formulæ. By an exercise of ingenuity, which, on the part of the first inventor, deserves to be regarded as considerable, two pairs of angles were hit upon, which united these requisites. First, it could be perceived intuitively that their differences were the angles at the base; and, secondly, they possessed one of the marks of equality, namely, coincidence when applied to one another. This coincidence, however, was not perceived intuitively, but inferred, in conformity to another formula.

For greater clearness, I subjoin an analysis of the demonstration. Euclid, it will be remembered, demonstrates his fifth proposition by means of the fourth. This it is not allowable for us to do, because we are undertaking to trace deductive truths not to prior deductions, but to their original inductive foundation. We must, therefore, use the premises of the fourth proposition instead of its conclusion, and prove the fifth directly from first principles. To do so requires six formulas. (We presuppose an equilateral triangle, whose vertices are A, D, E, with point B on the side AD, and point C on the side AE, such that BC is parallel to DE. We must begin, as in Euclid, by prolonging the equal sides AB, AC, to equal distances, and joining the extremities BE, DC.)

FIRST FORMULA.--*The sums of equals are equal.*

AD and AE are sums of equals by the supposition. Having that mark of equality, they are concluded by this formula to be equal.

SECOND FORMULA.--*Equal straight lines or angles, being applied to one another, coincide.*

AC, AB, are within this formula by supposition; AD, AE, have been brought within it by the preceding step. The angle at A considered as an angle of the triangle ABE, and the same angle considered as an angle of the triangle ACD, are of course within the formula. All these pairs, therefore, possess the property which, according to the second formula, is a mark that when applied to one another they will coincide. Conceive them, then, applied to one another, by turning over the triangle ABE, and laying it on the triangle ACD in such a manner that AB of the one shall lie upon AC of the other. Then, by the equality of the angles, AE will lie on AD. But AB and AC, AE and AD are equals; therefore they will coincide altogether, and of course at their extremities, D, E, and B, C.

THIRD FORMULA.--*Straight lines, having their extremities coincident, coincide.*

BE and CD have been brought within this formula by the preceding induction; they will, therefore, coincide.

FOURTH FORMULA.--*Angles, having their sides coincident, coincide.*

The third induction having shown that BE and CD coincide, and the second that AB, AC, coincide, the angles ABE and ACD are thereby brought within the fourth formula, and accordingly coincide.

FIFTH FORMULA.--*Things which coincide are equal.*

The angles ABE and ACD are brought within this formula by the induction immediately preceding. This train of reasoning being also applicable, *mutatis mutandis*, to the angles EBC, DCB, these also are brought within the fifth formula. And, finally,

SIXTH FORMULA.--*The differences of equals are equal.*

The angle ABC being the difference of ABE, CBE, and the angle ACB being the difference of ACD, DCB; which have been proved to be equals; ABC and ACB are brought within the last formula by the whole of the previous process.

The difficulty here encountered is chiefly that of figuring to ourselves the two angles at the base of the triangle ABC as remainders made by cutting one pair of angles out of another, while each pair shall be corresponding angles of triangles which have two sides and the intervening angle equal. It is by this happy contrivance that so many different inductions are brought to bear upon the same particular case. And this not being at all an obvious thought, it may be seen from an example so near the threshold of mathematics, how much scope there may well be for scientific dexterity in the higher branches of that and other sciences, in order so to combine a few simple inductions, as to bring within each of them innumerable cases which are not obviously included in it; and how long, and numerous, and complicated may be the processes necessary for bringing the inductions together, even when each induction may itself be very easy and simple. All the inductions involved in all geometry are comprised in those simple ones, the formulæ of which are the Axioms, and a few of the so-called Definitions. The remainder of the science is made up of the processes employed for bringing unforeseen cases within these inductions; or (in syllogistic language) for proving the minors necessary to complete the syllogisms; the majors being the definitions and axioms. In those definitions and axioms are laid down the whole of the marks, by an artful combination of which it has been found possible to discover and prove all that is proved in geometry. The marks being so few, and the inductions which furnish them being so obvious and familiar; the connecting of several of them together, which constitutes Deductions, or Trains of Reasoning, forms the whole difficulty of the science, and, with a trifling exception, its whole bulk; and hence Geometry is a Deductive Science.

§ 5. It will be seen hereafter(67) that there are weighty scientific reasons for giving to every science as much of the character of a Deductive Science as possible; for endeavoring to construct the science from the fewest and the simplest possible inductions, and to make these, by any combinations however complicated, suffice for proving even such truths, relating to complex cases, as could be proved, if we chose, by inductions from specific experience. Every branch of natural philosophy was originally experimental; each generalization rested on a special induction, and was derived from its own distinct set of observations and experiments. From being sciences of pure experiment, as the phrase is, or, to speak more correctly, sciences in which the reasonings mostly consist of no more than one step, and are expressed by single syllogisms, all these sciences have become to some extent, and some of them in nearly the whole of their extent, sciences of pure reasoning; whereby multitudes of truths, already known by induction from as many different sets of experiments, have come to be exhibited as deductions or corollaries from inductive propositions of a simpler and more universal character. Thus mechanics, hydrostatics, optics, acoustics, thermology, have successively been rendered mathematical; and astronomy was brought by Newton within the laws of general mechanics. Why it is that the substitution of this circuitous mode of proceeding for a process apparently much easier and more natural, is held, and justly, to be the greatest triumph of the investigation of nature, we are not, in this stage of our inquiry, prepared to examine. But it is necessary to remark, that although, by this progressive transformation, all sciences tend to become more and more Deductive, they are not, therefore, the less Inductive; every step in the Deduction is still an Induction. The opposition is not between the terms Deductive and Inductive, but between Deductive and Experimental. A science is experimental, in proportion as every new case, which presents any peculiar features, stands in need of a new set of observations and experiments--a fresh induction. It is deductive, in proportion as it can draw conclusions, respecting cases of a new kind, by processes which bring those cases under old inductions; by ascertaining that cases which can not be observed to have the requisite marks, have, however, marks of those marks.

We can now, therefore, perceive what is the generic distinction between sciences which can be made Deductive, and those which must as yet remain Experimental. The difference consists in our having been able, or not yet able, to discover marks of marks. If by our various inductions we have been able to proceed no further than to such propositions as these, *a* a mark of *b*, or *a* and *b* marks of one another, *c* a mark of *d*, or c and *d* marks of one another, without any thing to connect *a* or *b* with *c* or *d*; we have a science of detached and mutually independent generalizations, such as these, that acids redden vegetable blues, and that alkalies color them green; from neither of which propositions could we, directly or indirectly, infer the other: and a science, so far as it is composed of such propositions, is purely experimental. Chemistry, in the present state of our knowledge, has not yet thrown off this character. There are other sciences, however, of which the propositions are of this kind: *a* a mark of *b, b* a mark of *c, c* of *d, d* of *e*, etc. In these sciences we can mount the ladder from *a* to *e* by a process of ratiocination; we can conclude that *a* is a mark of *e*, and that every object which has the mark *a* has the property *e*, although, perhaps, we never were able to observe *a* and *e* together, and although even *d*, our only direct mark of *e*, may not be perceptible in those objects, but only inferable. Or, varying the first metaphor, we may be said to get from *a* to *e* underground: the marks *b*, *c*, *d*, which indicate the route, must all be possessed somewhere by the objects concerning which we are inquiring; but they are below the surface: *a* is the only mark that is visible, and by it we are able to trace in succession all the rest.

§ 6. We can now understand how an experimental may transform itself into a deductive science by the mere progress of experiment. In an experimental science, the inductions, as we have said, lie detached, as, *a* a mark of *b, c* a mark of *d*, *e* a mark of *f*, and so on: now, a new set of instances, and a consequent new induction, may at any time bridge over the interval between two of these unconnected arches; *b*, for example, may be ascertained to be a mark of *c*, which enables us thenceforth to prove deductively that *a* is a mark of *c*. Or, as sometimes happens, some comprehensive induction may raise an arch high in the air, which bridges over hosts of them at once; *b*, *d*, *f*, and all the rest, turning out to be marks of some one thing, or of things between which a connection has already been traced. As when Newton discovered that the motions, whether regular or apparently anomalous, of all the bodies of the solar system (each of which motions had been inferred by a separate logical operation, from separate marks), were all marks of moving round a common centre, with a centripetal force varying directly as the mass, and inversely as the square of the distance from that centre. This is the greatest example which has yet occurred of the transformation, at one stroke, of a science which was still to a great degree merely experimental, into a deductive science.

Transformations of the same nature, but on a smaller scale, continually take place in the less advanced branches of physical knowledge, without enabling them to throw off the character of experimental sciences. Thus with regard to the two unconnected propositions before cited, namely, Acids redden vegetable blues, Alkalies make them green; it is remarked by Liebig, that all blue coloring matters which are reddened by acids (as well as, reciprocally, all red coloring matters which are rendered blue by alkalies) contain nitrogen: and it is quite possible that this circumstance may one day furnish a bond of connection between the two propositions in question, by showing that the antagonistic action of acids and alkalies in producing or destroying the color blue, is the result of some one, more general, law. Although this connecting of detached generalizations is so much gain, it tends but little to give a deductive character to any science as a whole; because the new courses of observation and experiment, which thus enable us to connect together a few general truths, usually make known to us a still greater number of unconnected new ones. Hence chemistry, though similar extensions and simplifications of its generalizations are continually taking place, is still in the main an experimental science; and is likely so to continue unless some comprehensive induction should be hereafter arrived at, which, like Newton's, shall connect a vast number of the smaller known inductions together, and change the whole method of the science at once. Chemistry has already one great generalization, which, though relating to one of the subordinate aspects of chemical phenomena, possesses within its limited sphere this comprehensive character; the principle of Dalton, called the atomic theory, or the doctrine of chemical equivalents: which by enabling us to a certain extent to foresee the proportions in which two substances will combine, before the experiment has been tried, constitutes undoubtedly a source of new chemical truths obtainable by deduction, as well as a connecting principle for all truths of the same description previously obtained by experiment.

§ 7. The discoveries which change the method of a science from experimental to deductive, mostly consist in establishing, either by deduction or by direct experiment, that the varieties of a particular phenomenon uniformly accompany the varieties of some other phenomenon better known. Thus the science of sound, which previously stood in the lowest rank of merely experimental science, became deductive when it was proved by experiment that every variety of sound was consequent on, and therefore a mark of, a distinct and definable variety of oscillatory motion among the particles of the transmitting medium. When this was ascertained, it followed that every relation of succession or co-existence which obtained between phenomena of the more known class, obtained also between the phenomena which correspond to them in the other class. Every sound, being a mark of a particular oscillatory motion, became a mark of every thing which, by the laws of dynamics, was known to be inferable from that motion; and every thing which by those same laws was a mark of any oscillatory motion among the particles of an elastic medium, became a mark of the corresponding sound. And thus many truths, not before suspected, concerning sound, become deducible from the known laws of the propagation of motion through an elastic medium; while facts already empirically known respecting sound, become an indication of corresponding properties of vibrating bodies, previously undiscovered.

But the grand agent for transforming experimental into deductive sciences, is the science of number. The properties of number, alone among all known phenomena, are, in the most rigorous sense, properties of all things whatever. All things are not colored, or ponderable, or even extended; but all things are numerable. And if we consider this science in its whole extent, from common arithmetic up to the calculus of variations, the truths already ascertained seem all but infinite, and admit of indefinite extension.

These truths, though affirmable of all things whatever, of course apply to them only in respect of their quantity. But if it comes to be discovered that variations of quality in any class of phenomena, correspond regularly to variations of quantity either in those same or in some other phenomena; every formula of mathematics applicable to quantities which vary in that particular manner, becomes a mark of a corresponding general truth, respecting the variations in quality which accompany them: and the science of quantity being (as far as any science can be) altogether deductive, the theory of that particular kind of qualities becomes, to this extent, deductive likewise.

The most striking instance in point which history affords (though not an example of an experimental science rendered deductive, but of an unparalleled extension given to the deductive process in a science which was deductive already), is the revolution in geometry which originated with Descartes, and was completed by Clairaut. These great mathematicians pointed out the importance of the fact, that to every variety of position in points, direction in lines, or form in curves or surfaces (all of which are Qualities), there corresponds a peculiar relation of quantity between either two or three rectilineal co-ordinates; insomuch that if the law were known according to which those co-ordinates vary relatively to one another, every other geometrical property of the line or surface in question, whether relating to quantity or quality, would be capable of being inferred. Hence it followed that every geometrical question could be solved, if the corresponding algebraical one could; and geometry received an accession (actual or potential) of new truths, corresponding to every property of numbers which the progress of the calculus had brought, or might in future bring, to light. In the same general manner, mechanics, astronomy, and in a less degree, every branch of natural philosophy commonly so called, have been made algebraical. The varieties of physical phenomena with which those sciences are conversant, have been found to answer to determinable varieties in the quantity of some circumstance or other; or at least to varieties of form or position, for which corresponding equations of quantity had already been, or were susceptible of being, discovered by geometers.

In these various transformations, the propositions of the science of number do but fulfill the function proper to all propositions forming a train of reasoning, viz., that of enabling us to arrive in an indirect method, by marks of marks, at such of the properties of objects as we can not directly ascertain (or not so conveniently) by experiment. We travel from a given visible or tangible fact, through the truths of numbers, to the facts sought. The given fact is a mark that a certain relation subsists between the quantities of some of the elements concerned; while the fact sought presupposes a certain relation between the quantities of some other elements: now, if these last quantities are dependent in some known manner upon the former, or *vicè versa*, we can argue from the numerical relation between the one set of quantities, to determine that which subsists between the other set; the theorems of the calculus affording the intermediate links. And thus one of the two physical facts becomes a mark of the other, by being a mark of a mark of a mark of it.