Chapter XV.

Of Progressive Effects; And Of The Continued Action Of Causes.

§ 1. In the last four chapters we have traced the general outlines of the theory of the generation of derivative laws from ultimate ones. In the present chapter our attention will be directed to a particular case of the derivation of laws from other laws, but a case so general, and so important as not only to repay, but to require, a separate examination. This is the case of a complex phenomenon resulting from one simple law, by the continual addition of an effect to itself.

There are some phenomena, some bodily sensations, for example, which are essentially instantaneous, and whose existence can only be prolonged by the prolongation of the existence of the cause by which they are produced. But most phenomena are in their own nature permanent; having begun to exist, they would exist forever unless some cause intervened having a tendency to alter or destroy them. Such, for example, are all the facts of phenomena which we call bodies. Water, once produced, will not of itself relapse into a state of hydrogen and oxygen; such a change requires some agent having the power of decomposing the compound. Such, again, are the positions in space and the movements of bodies. No object at rest alters its position without the intervention of some conditions extraneous to itself; and when once in motion, no object returns to a state of rest, or alters either its direction or its velocity, unless some new external conditions are superinduced. It, therefore, perpetually happens that a temporary cause gives rise to a permanent effect. The contact of iron with moist air for a few hours, produces a rust which may endure for centuries; or a projectile force which launches a cannon-ball into space, produces a motion which would continue forever unless some other force counteracted it.

Between the two examples which we have here given, there is a difference worth pointing out. In the former (in which the phenomenon produced is a substance, and not a motion of a substance), since the rust remains forever and unaltered unless some new cause supervenes, we may speak of the contact of air a hundred years ago as even the proximate cause of the rust which has existed from that time until now. But when the effect is motion, which is itself a change, we must use a different language. The permanency of the effect is now only the permanency of a series of changes. The second foot, or inch, or mile of motion is not the mere prolonged duration of the first foot, or inch, or mile, but another fact which succeeds, and which may in some respects be very unlike the former, since it carries the body through a different region of space. Now, the original projectile force which set the body moving is the remote cause of all its motion, however long continued, but the proximate cause of no motion except that which took place at the first instant. The motion at any subsequent instant is proximately caused by the motion which took place at the instant preceding. It is on that, and not on the original moving cause, that the motion at any given moment depends. For, suppose that the body passes through some resisting medium, which partially counteracts the effect of the original impulse, and retards the motion; this counteraction (it need scarcely here be repeated) is as strict an example of obedience to the law of the impulse, as if the body had gone on moving with its original velocity; but the motion which results is different, being now a compound of the effects of two causes acting in contrary directions, instead of the single effect of one cause. Now, what cause does the body obey in its subsequent motion? The original cause of motion, or the actual motion at the preceding instant? The latter; for when the object issues from the resisting medium, it continues moving, not with its original, but with its retarded velocity. The motion having once been diminished, all that which follows is diminished. The effect changes, because the cause which it really obeys, the proximate cause, the real cause in fact, has changed. This principle is recognized by mathematicians when they enumerate among the causes by which the motion of a body is at any instant determined the *force generated* by the previous motion; an expression which would be absurd if taken to imply that this "force" was an intermediate link between the cause and the effect, but which really means only the previous motion itself, considered as a cause of further motion. We must, therefore, if we would speak with perfect precision, consider each link in the succession of motions as the effect of the link preceding it. But if, for the convenience of discourse, we speak of the whole series as one effect, it must be as an effect produced by the original impelling force; a permanent effect produced by an instantaneous cause,

and possessing the property of self-perpetuation.

Let us now suppose that the original agent or cause, instead of being instantaneous, is permanent. Whatever effect has been produced up to a given time, would (unless prevented by the intervention of some new cause) subsist permanently, even if the cause were to perish. Since, however, the cause does not perish, but continues to exist and to operate, it must go on producing more and more of the effect; and instead of a uniform effect, we have a progressive series of effects, arising from the accumulated influence of a permanent cause. Thus, the contact of iron with the atmosphere causes a portion of it to rust; and if the cause ceased, the effect already produced would be permanent, but no further effect would be added. If, however, the cause, namely, exposure to moist air, continues, more and more of the iron becomes rusted, until all which is exposed is converted into a red powder, when one of the conditions of the production of rust, namely, the presence of unoxidized iron, has ceased, and the effect can not any longer be produced. Again, the earth causes bodies to fall toward it; that is, the existence of the earth at a given instant causes an unsupported body to move toward it at the succeeding instant; and if the earth were annihilated, as much of the effect as is already produced would continue; the object would go on moving in the same direction, with its acquired velocity, until intercepted by some body or deflected by some other force. The earth, however, not being annihilated, goes on producing in the second instant an effect similar and of equal amount with the first, which two effects being added together, there results an accelerated velocity; and this operation being repeated at each successive instant, the mere permanence of the cause, though without increase, gives rise to a constant progressive increase of the effect, so long as all the conditions, negative and positive, of the production of that effect continue to be realized.

It is obvious that this state of things is merely a case of the Composition of Causes. A cause which continues in action must on a strict analysis be considered as a number of causes exactly similar, successively introduced, and producing by their combination the sum of the effects which they would severally produce if they acted singly. The progressive rusting of the iron is in strictness the sum of the effects of many particles of air acting in succession upon corresponding particles of iron. The continued action of the earth upon a falling body is equivalent to a series of forces, applied in successive instants, each tending to produce a certain constant quantity of motion; and the motion at each instant is the sum of the effects of the new force applied at the preceding instant, and the motion already acquired. In each instant a fresh effect, of which gravity is the proximate cause, is added to the effect of which it was the remote cause; or (to express the same thing in another manner), the effect produced by the earth's influence at the instant last elapsed is added to the sum of the effects of which the remote causes were the influences exerted by the earth at all the previous instants since the motion began. The case, therefore, comes under the principle of a concurrence of causes producing an effect equal to the sum of their separate effects. But as the causes come into play not all at once, but successively, and as the effect at each instant is the sum of the effects of those causes only which have come into action up to that instant, the result assumes the form of an ascending series; a succession of sums, each greater than that which preceded it; and we have thus a progressive effect from the continued action of a cause.

Since the continuance of the cause influences the effect only by adding to its quantity, and since the addition takes place according to a fixed law (equal quantities in equal times), the result is capable of being computed on mathematical principles. In fact, this case, being that of infinitesimal increments, is precisely the case which the differential calculus was invented to meet. The questions, what effect will result from the continual addition of a given cause to itself, and what amount of the cause, being continually added to itself, will produce a given amount of the effect, are evidently mathematical questions, and to be treated, therefore, deductively. If, as we have seen, cases of the Composition of Causes are seldom adapted for any other than deductive investigation, this is especially true in the case now examined, the continual composition of a cause with its own previous effects; since such a case is peculiarly amenable to the deductive method, while the undistinguishable manner in which the effects are blended with one another and with the causes, must make the treatment of such an instance experimentally still more chimerical than in any other case.

§ 2. We shall next advert to a rather more intricate operation of the same principle, namely, when the cause

does not merely continue in action, but undergoes, during the same time, a progressive change in those of its circumstances which contribute to determine the effect. In this case, as in the former, the total effect goes on accumulating by the continual addition of a fresh effect to that already produced, but it is no longer by the addition of equal quantities in equal times; the quantities added are unequal, and even the quality may now be different. If the change in the state of the permanent cause be progressive, the effect will go through a double series of changes, arising partly from the accumulated action of the cause, and partly from the changes in its action. The effect is still a progressive effect, produced, however, not by the mere continuance of a cause, but by its continuance and its progressiveness combined.

A familiar example is afforded by the increase of the temperature as summer advances, that is, as the sun draws nearer to a vertical position, and remains a greater number of hours above the horizon. This instance exemplifies in a very interesting manner the twofold operation on the effect, arising from the continuance of the cause, and from its progressive change. When once the sun has come near enough to the zenith, and remains above the horizon long enough, to give more warmth during one diurnal rotation than the counteracting cause, the earth's radiation, can carry off, the mere continuance of the cause would progressively increase the effect, even if the sun came no nearer and the days grew no longer; but in addition to this, a change takes place in the accidents of the cause (its series of diurnal positions), tending to increase the quantity of the effect. When the summer solstice has passed, the progressive change in the cause begins to take place the reverse way, but, for some time, the accumulating effect of the mere continuance of the cause exceeds the effect of the changes in it, and the temperature continues to increase.

Again, the motion of a planet is a progressive effect, produced by causes at once permanent and progressive. The orbit of a planet is determined (omitting perturbations) by two causes: first, the action of the central body, a permanent cause, which alternately increases and diminishes as the planet draws nearer to or goes farther from its perihelion, and which acts at every point in a different direction; and, secondly, the tendency of the planet to continue moving in the direction and with the velocity which it has already acquired. This force also grows greater as the planet draws nearer to its perihelion, because as it does so its velocity increases, and less, as it recedes from its perihelion; and this force as well as the other acts at each point in a different direction, because at every point the action of the central force, by deflecting the planet from its previous direction, alters the line in which it tends to continue moving. The motion at each instant is determined by the amount and direction of the motion, and the amount and direction of the sun's action, at the previous instant; and if we speak of the entire revolution of the planet as one phenomenon (which, as it is periodical and similar to itself, we often find it convenient to do), that phenomenon is the progressive effect of two permanent and progressive causes, the central force and the acquired motion. Those causes happening to be progressive in the particular way which is called periodical, the effect necessarily is so too; because the quantities to be added together returning in a regular order, the same sums must also regularly return.

This example is worthy of consideration also in another respect. Though the causes themselves are permanent, and independent of all conditions known to us, the changes which take place in the quantities and relations of the causes are actually caused by the periodical changes in the effects. The causes, as they exist at any moment, having produced a certain motion, that motion, becoming itself a cause, reacts upon the causes, and produces a change in them. By altering the distance and direction of the central body relatively to the planet, and the direction and quantity of the force in the direction of the tangent, it alters the elements which determine the motion at the next succeeding instant. This change renders the next motion somewhat different; and this difference, by a fresh reaction upon the causes, renders the next motion again different, and so on. The original state of the causes might have been such that this series of actions modified by reactions would not have been periodical. The sun's action, and the original impelling force, might have been in such a ratio to one another, that the reaction of the effect would have been such as to alter the causes more and more, without ever bringing them back to what they were at any former time. The planet would then have moved in a parabola, or an hyperbola, curves not returning into themselves. The quantities of the two forces were, however, originally such, that the successive reactions of the effect bring back the causes, after a certain time, to what they were before; and from that time all the variations continued to recur again and again in the same

periodical order, and must so continue while the causes subsist and are not counteracted.

§ 3. In all cases of progressive effects, whether arising from the accumulation of unchanging or of changing elements, there is a uniformity of succession not merely between the cause and the effect, but between the first stages of the effect and its subsequent stages. That a body *in vacuo* falls sixteen feet in the first second, forty-eight in the second, and so on in the ratio of the odd numbers, is as much a uniform sequence as that when the supports are removed the body falls. The sequence of spring and summer is as regular and invariable as that of the approach of the sun and spring; but we do not consider spring to be the cause of summer; it is evident that both are successive effects of the heat received from the sun, and that, considered merely in itself, spring might continue forever without having the slightest tendency to produce summer. As we have so often remarked, not the conditional, but the unconditional invariable antecedent is termed the cause. That which would not be followed by the effect unless something else had preceded, and which if that something else had preceded, would not have been required, is not the cause, however invaluable the sequence may in fact be.

It is in this way that most of those uniformities of succession are generated, which are not cases of causation. When a phenomenon goes on increasing, or periodically increases and diminishes, or goes through any continued and unceasing process of variation reducible to a uniform rule or law of succession, we do not on this account presume that any two successive terms of the series are cause and effect. We presume the contrary; we expect to find that the whole series originates either from the continued action of fixed causes or from causes which go through a corresponding process of continuous change. A tree grows from half an inch high to a hundred feet; and some trees will generally grow to that height unless prevented by some counteracting cause. But we do not call the seedling the cause of the full-grown tree; the invariable antecedent it certainly is, and we know very imperfectly on what other antecedents the sequence is contingent, but we are convinced that it is contingent on something; because the homogeneousness of the antecedent with the consequent, the close resemblance of the seedling to the tree in all respects except magnitude, and the graduality of the growth, so exactly resembling the progressively accumulating effect produced by the long action of some one cause, leave no possibility of doubting that the seedling and the tree are two terms in a series of that description, the first term of which is yet to seek. The conclusion is further confirmed by this, that we are able to prove by strict induction the dependence of the growth of the tree, and even of the continuance of its existence, upon the continued repetition of certain processes of nutrition, the rise of the sap, the absorptions and exhalations by the leaves, etc.; and the same experiments would probably prove to us that the growth of the tree is the accumulated sum of the effects of these continued processes, were we not, for want of sufficiently microscopic eyes, unable to observe correctly and in detail what those effects are.

This supposition by no means requires that the effect should not, during its progress, undergo many modifications besides those of quantity, or that it should not sometimes appear to undergo a very marked change of character. This may be either because the unknown cause consists of several component elements or agents, whose effects, accumulating according to different laws, are compounded in different proportions at different periods in the existence of the organized being; or because, at certain points in its progress, fresh causes or agencies come in, or are evolved, which intermix their laws with those of the prime agent.